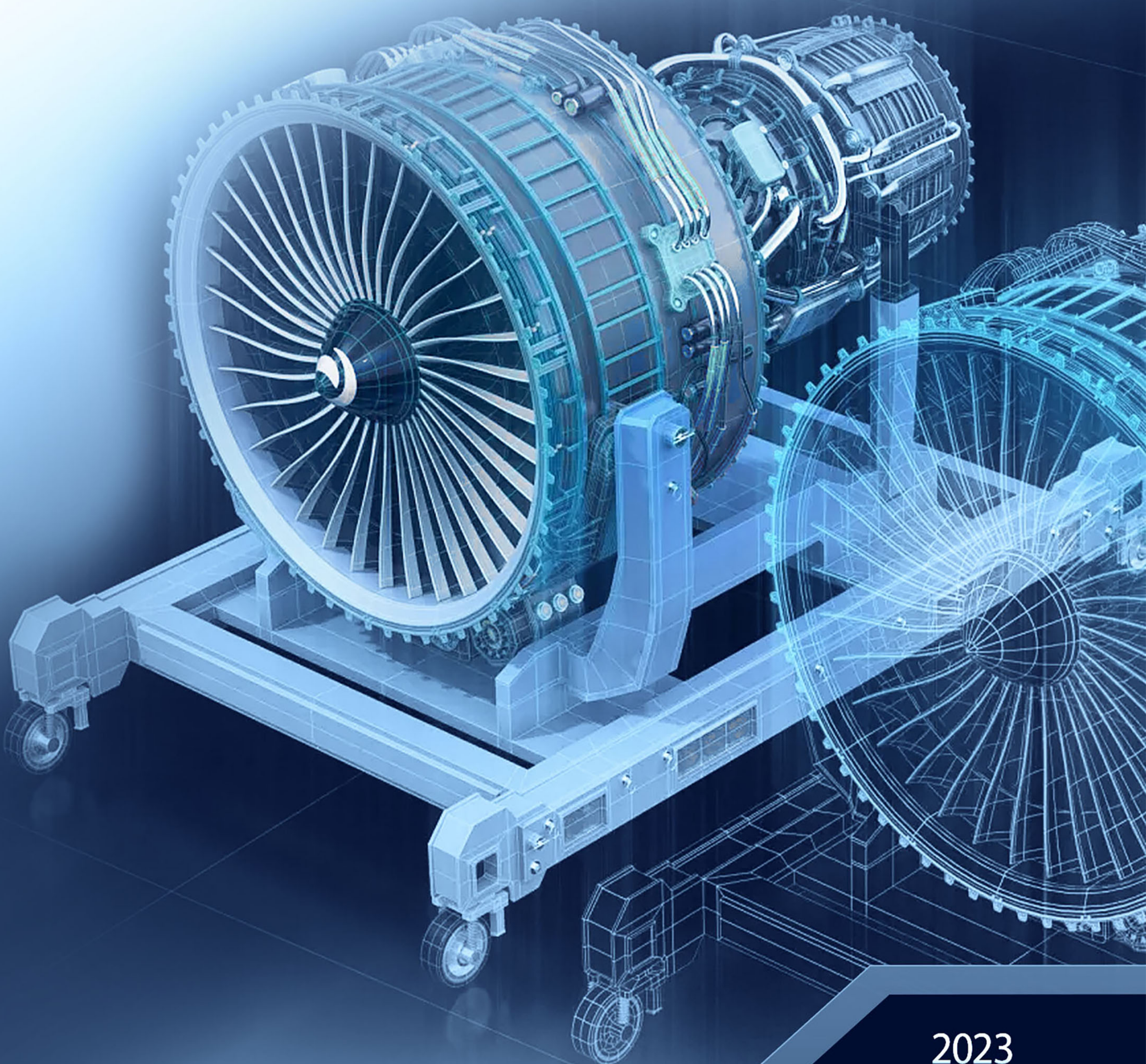
PETRENKO O. M., SHAVKUN V. M.,
DONETS O.V., ZUBENKO D.Y.

STUDY OF STEADY-STATE MODES OF A SYNCHRONOUS ELECTRIC MACHINE

MONOGRAPH



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The monograph contains theoretical studies and a mathematical apparatus justifying the use of synchronous permanent magnet motors. Comparative calculations of synchronous machines and permanent magnet motors are shown.
The monograph is intended for specialists of design, transport and municipal organizations of the urban economy, as well as for teachers, graduate students and students of technical specialties.

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Content

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Introduction / Вступ

Unlike three-phase asynchronous motors, permanent magnet motors do not have a rotor winding, but, as their name implies, are equipped with permanent magnets. In particular, to take the simplest case, the stator has the same shape as an induction motor. Manufacturers of this type of motor are also working to optimize the design of the devices [1].

Permanent magnet motors are synchronous, which means that there is no slip between the rotating fields of the rotor and stator, which distinguishes them from three- phase asynchronous motors. Permanent magnets provide the necessary rotor magnetization without corresponding losses, which increases the efficiency of this type of motor compared to an asynchronous motor. This technology has been used for a long time for the production of servo drives. Now the size of the device complies with the IEC standard. Because magnets require expensive materials to produce, the price of such motors was very high until recently, and demand far exceeded supply. However, over the past two years, there has been a significant decline in prices. This is partly due to the discovery of new sources of the necessary raw materials [2].

In reduced speed operation, permanent magnet motors are more efficient than asynchronous motors in all operating modes due to lower power losses. In practice, a modern permanent magnet motor achieves an efficiency class of IE3 to IE4.

Compared to an asynchronous motor of a similar efficiency class, such as the IE3, the overall size of a permanent magnet motor is half that of a standard motor. This type of motor can be operated using a frequency converter alone, provided that it is equipped with an appropriate control system. Indeed, the operation of a permanent magnet motor is carried out using an electronic controller. The types of motors with load starting have short-circuited rotors. This damping effect has a negative impact on the startup and efficiency of the motor when operating with a frequency converter [3]. A significant disadvantage of permanent magnet motors is the need for a frequency converter or controller. The controller must also receive a positional feedback signal in order to optimally adapt the magnetic field and generate rotation. This is why such systems are often equipped with an encoder. Nevertheless, some manufacturers (including Danfoss) offer technical solutions that allow you to control this type of motor without using an encoder [4].

The other two disadvantages of these motors include demagnetization at high current and temperature values, which, however, is rarely encountered in practice; and problems associated with motor repair. Due to the presence of strong magnets in the rotor, the process of removing the rotor from the stator is complicated and requires the use of special tools [5].

CHAPTER 1
TYPES AND PRINCIPLES OF OPERATION, WITH PERMANENT MAGNETS

1.1. Designs and types of synchronous electric motors with permanent magnets

A permanent magnet synchronous motor (*PMSM*) is a synchronous electric motor whose inductor consists of permanent magnets.

The main difference between a permanent magnet synchronous motor (PMSM) and an induction motor is the rotor. Studies conducted by1 show that a SSCM has an efficiency of about 2% higher than a highly efficient (IE3) induction motor, provided that the stator has the same design and the same frequency converter is used for control [6].

A permanent magnet synchronous motor (like any rotating electric motor) consists of a rotor and a stator. The stator is the stationary part, and the rotor is the rotating part [7].

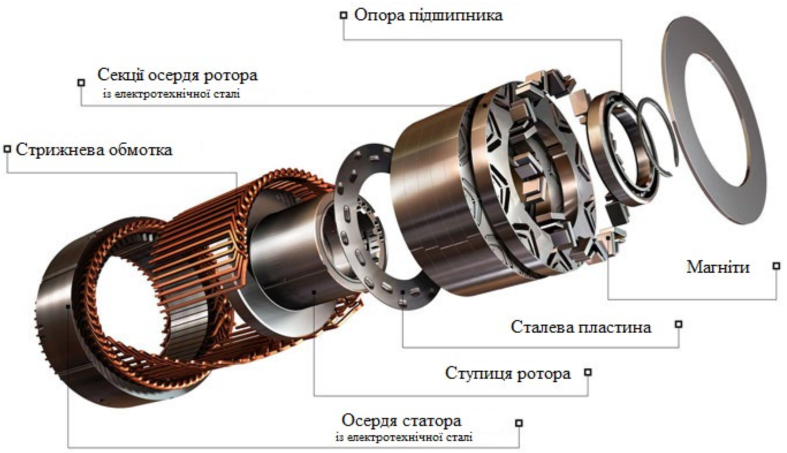

Figure 1.1 - Synchronous motor with built-in permanent magnets

Usually, the rotor is located in the stator bore of the electric motor, but there are designs with an external rotor - inverted-type electric motors [9].

The rotor consists of permanent magnets. Materials with high coercive force are used as permanent magnets [10].

According to the rotor design, synchronous motors are divided into:

- electric motors with pronounced poles;
- electric motors with implicit poles.

A motor with implicit poles has equal inductance along the longitudinal and transverse axes $L_d = L_q$whereas the motor with explicit poles has transverse inductance not equal to the longitudinal $L_q \neq L_d$.

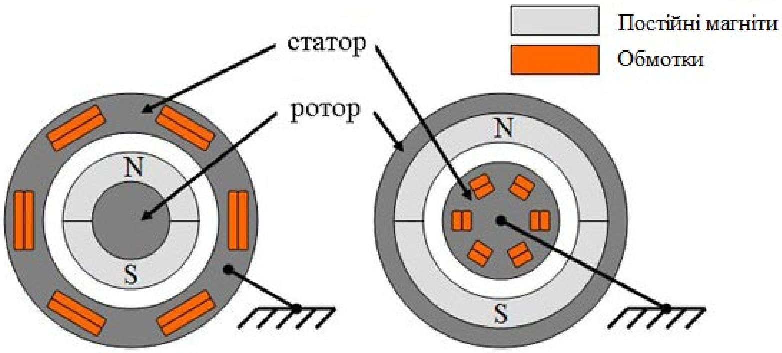

Figure 1.2 - Designs of permanent magnet synchronous motors:
left - standard, right - reversed.

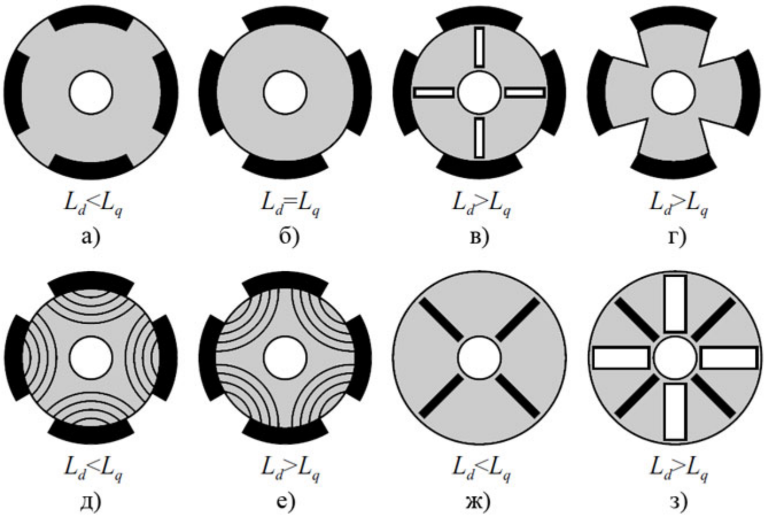

Figure 1.3 - Cross-section of rotors with different ratios Ld/Lq
Magnets are marked in black.

Figures *d* and *e* show axially stratified rotors, and Figures *c* and *h* show rotors with barriers.

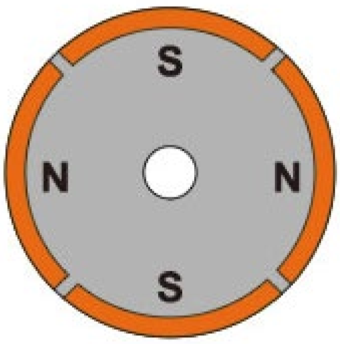

Figure 1.4 - Rotor of a synchronous motor with a surface-mounted permanent magnet installation.

Also, according to the design of the rotor, SPDMs are divided into:

- synchronous motor with a surface permanent magnet installation (SPMSM - surface permanent magnet synchronous motor);

- synchronous motor **with built-in** (incorporated) magnets (IPMSM - internal permanent magnet synchronous motor).

Figure 1.5 - Synchronous motor rotor with built-in magnets

The stator consists of a body and a pole with bushings. The most common designs are two- and three-phase windings [11].

Depending on the design of the stator, a permanent magnet synchronous motor can be

- with distributed winding;
- with a concentric winding.

A distributed winding is a winding in which the number of slots per pole and phase $Q = 2, 3, , k$.

A winding is called **concentrated if the** number of slots per pole and phase $Q = 1$. In this case, the slots are evenly spaced around the stator circumference. The two coils that form the winding can be connected either in series or in parallel [12]. The main disadvantage of such windings is the inability to influence the shape of the EMF curve [2].

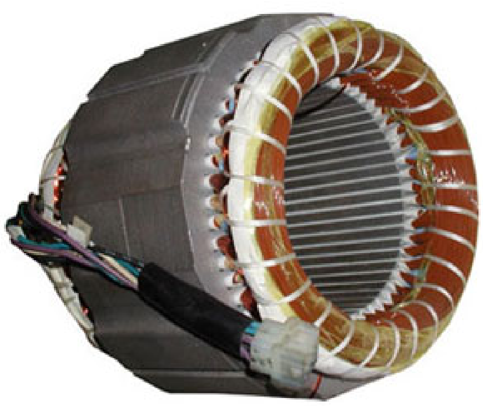

Figure 1.6 - Electric motor stator with distributed winding

Figure 1.7 - Motor stator with concentric winding

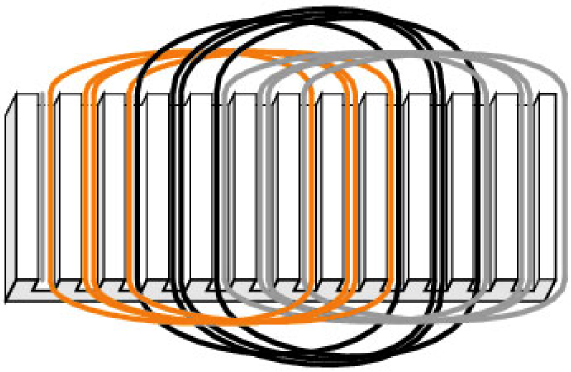

Figure 1.8 - Diagram of a three-phase distributed winding

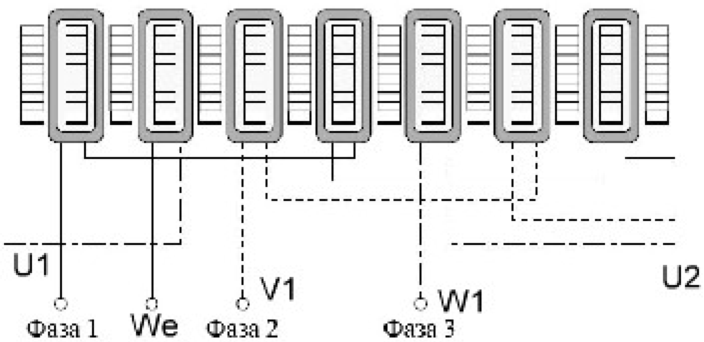

Figure 1.9 - Diagram of a three-phase concentrated winding

The form of the inverse EMF of an electric motor can be:

- trapezoidal;
- sinusoidal.

The shape of the EMF curve in the conductor is determined by the distribution curve of magnetic induction in the gap around the stator circumference.

It is known that the magnetic induction in the gap under a pronounced rotor pole has a trapezoidal shape. The EMF induced in a conductor has the same shape. If it is necessary to create a sinusoidal EMF, then the pole tip is shaped in such a way that the induction distribution curve would be close to sinusoidal [13]. This is facilitated by the bevels of the rotor pole tips [2].

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CHAPTER 2
VECTOR DIAGRAMS AND EQUASIONS OF STEADY-STATE MODES OF THE SYNCHRONOUS MACHINE

2.1. Vector diagram of a nonsalient pole synchronous generator

To analyze the work of electric machines with permanent magnets, we will construct spatial and temporal vector diagrams of E. M. F. of the synchronous generator in the received symbols. The direction of the rotor spinning is assumed to coincide with the direction of the vectors rotation in the temporal diagram, that is, counterclockwise. The position of the rotor is shown at the moment when in the phase U1-U2: the current has the maximum positive value. As is known from the basic course of electric machines, the three-phase current of a direct sequence, passing through a three-phase winding, creates a rotating with a synchronous speed constant according to the magnitude of m. f. of the armature reaction $F_m$whose value is 3/2 times higher than the maximum value of m. f. of one phase [14]. At the same time, at the moment when the current in any of the phases has the maximum value of rotating, the m. f. of armature reaction coincides with the direction of m. f. of this phase. Consequently, the vector F_m of m. f. of the armature reaction should be directed vertically upwards.

E. M. F. *E.* in the phase U1, induced by the flow of the rotor Φ_f, had a maximum value at the moment when the pole axis coincided with the phase plane. Then the elongated axis of pole *d* turns to the angle ψ of electric degrees and only then the current in phase ***a*** reaches a maximum [15]. Therefore, under the present conditions, the current in stator *I* reveals a phase lag behind E. R. S. *E* by the angle ψ, equal to the angle between the axis of the rotor *d* and the plane of phase U1. If, by combining the temporal diagram of flows, currents, and E. M. F. with the spatial diagram of M. F. of flows, equally direct the common for both diagrams the vector of the rotor flow Φ_f, then the vector of m. f. of the armature reaction F_m coincides with the current vector *I* (the phase in which the latter is at the moment the maximum, that is, with current) in the phase U1. Since, (in addition), both of these vectors rotate with the same synchronous velocity, then their coincidence will continue at all subsequent moments of time [16].

Vectors of M.F. and the excitation winding flow F_f and Φ_f have the same direction, which coincides with the positive direction of the longitudinal axis *d*. Thus, in the space the angle between the vectors of m. f. of armature reactions F_m and m. f. of the excitation winding F_f is 90 + ψ e. degrees. The geometric sum of m. f. vectors of the excitation winding and the armature reaction has a direction depending on the ratio of absolute values of the given m. f. and from the angle ψ between them. The latter alters with the change in the nature of the load. In nonsalient pole machines, the magnetic resistance is minimal in the direction of the longitudinal axis *d* and maximal on the lateral axis *q*, so that when changing the nature of the load to avoid different magnetic resistances, the m. f. of the armature reaction is decomposed into the longitudinal F_{md} and lateral F_{mq} components. It follows that

$$F_{md} = F_m \cos(90° + \psi) = -F_m \sin\varphi \quad (2.1)$$

$$F_{mq} = F_m \cos(180° + \psi) = -F_m\, con\, \varphi, \quad (2.2)$$

where $90° + \psi$ is the angle between the vectors F_m and the positive direction of the longitudinal axis d;

$180° + \psi$ is the angle between the vectors F_m and the positive direction of the longitudinal axis q.

$$I_d = -I sin\, \psi, \quad (2.3)$$

$$I_q = -I cos\psi. \quad (2.4)$$

The sign - in formulas (2.1) - (2.4) suggests that the longitudinal and lateral components of the current and M. of the armature reactions will have negative values in case when the latter are directed toward the negative directions respective the longitudinal and lateral axes d and q. The angle ψ with the inductive nature of the load should be taken with a plus sign, and at a capacitive one with a minus sign [17].

This is especially emphasized by the fact that some authors [18], following R.G. Park, lateral and longitudinal components of the current and M. F. of the armature reactions are considered positive when they are directed according to the corresponding axes to the negative side. In this case, in formulas (2.3) and (2.4) they get a plus sign on the right side. Longitudinal and lateral m. f. of the armature reaction F_{md} and F_{mq} create, respectively, the longitudinal and lateral flows of the armature reaction and diffusion flows that induce in the stator winding the longitudinal and lateral E. M. F. of the complete armature reaction E_{ad} and E_{aq}. Effective values of these E. M. F. are considered to be taken with the opposite sign of the voltage drop in synchronous inductive resistances x_d and x_q respective the longitudinal and lateral axes, that is, it is assumed that

$$\overline{E_{ad}} = -jx_d\overline{I_d},$$
$$\overline{E_{aq}} = -jx_q\overline{I_q} \quad (2.5)$$

where I_d and I_q are the effective values of the longitudinal and lateral components of the stator current:

E. M. F. E_{ad} and E_{aq} reveal a phase lag behind the currents I_d and I_q at an angle of 90 °, and therefore the right side of formulas (2.5) is multiplied by-j.

Similarly, one can accept the E. M. F. $E_{\text{г}}$ vector equal and opposite to the voltage drop in the active resistance of the stator phase, that is, one can put it as follows:

$$E_{\text{г}} = -I_r \quad (2.6)$$

Arranging geometrically the vectors E. M. F. E. E_{ad}, E_{aq}, and $E_{\text{г}}$, we obtain the voltage vector of the generator $U_{\text{г}}$ (2.1), that is,

$$U_{\text{г}} = E + E_{ad} + E_{aq} + E_r \tag{2.7}$$

or, taking into account (2.5) and (2.6),

$$U_{\text{г}} = E - jx_d I_d - jx_q I_q - I_r. \tag{2.8}$$

The voltage vector of the network U, which the generator is connected to, is equal according to the magnitude and is opposite to the generator voltage according to the direction, that is

$$U = -E + jx_d I_d + jx_q I_q + I_r \tag{2.9}$$

The resulting voltage vector of the network U can be decomposed into two components $U_d$and $U_q$directed on the longitudinal and lateral axes of the rotor [18]. It immediately follows that

$$U_d = Ucos(90° - \delta) = U\sin\delta$$
$$U_q = U\cos\delta \tag{2.10}$$

where δ is the angle at which the voltage of the machine operating in this case as a generator lags behind E. M. F. E.

2.2. Vector diagram of a nonsalient pole synchronous motor

The vector diagram of the synchronous motor for the moment when the current in the phase U has the maximum value. In this case, the spatial and temporal diagrams are combined in exactly the same way as in the vector diagram of the synchronous generator discussed above that is, the vector of the rotor flow $\Phi_f$in both diagrams coincides [19]. The rotor flow created by a constant magnet is induced in the stator winding E.R. S. E, which lags behind this flow by 90 °. The current in the motor has a direction approximately opposite to E. M. F. Depending on whether the synchronous motor is overexcited or underexcited, the current will reveal a phase lag behind E. M. F. by the angle that is, respectively, slightly smaller or greater than 180°.

The case of overexcitation is taken when the synchronous motor is fed from the network by capacitive current, ahead of the network voltage and lags behind the phase from E. M. F. by the angle φ that is less than 180 °, but greater than 90 °.

Using the rule of the right hand, one can determine the direction of E. M. F. in the phase U. In this case, E. M. F.E. has a direction from the end of the phase to the beginning. The current in the motor will have a direction opposite to E. M. F., i.e., from the beginning to the end of the phase. In this case, as we arranged in Sections 2 and 1, the direction of m. f. of the phase ***a*** will be negative, that is from the center downward.

Since the diagram is plotted for the moment of time when the current in phase ***a*** has the maximum value, then m. f. of the armature reaction F_m will coincide with the negative direction of the axis of the given phase [20].

As in the case of the vector diagram of generator, we decompose the vectors of m. f. F_m and the current I into the longitudinal and lateral components. The longitudinal components of m. f. of both the armature and current reactions are directed toward the negative direction of the axis d and therefore have negative values in this case.

We deduce:

$$F_{md} = -F_m \sin\psi \tag{2.11}$$

$$I_d = -Isin\psi \tag{2.12}$$

$$F_{md} = -Fm \cos\psi \tag{2.13}$$

$$I_q = -I \cos\psi \tag{2.14}$$

As can be seen from the comparison (2.1) - (2.4) of (3.1) - (3.4) formulas for the longitudinal and lateral components of the current and m. f. of the armature reaction in the case of the generator and the motor are absolutely identical [21].

Longitudinal and lateral m. f. of the armature reaction creates, respectively, the longitudinal and lateral complete flows of the armature reaction, which induce in the stator winding e. m. f. E_{ad} and E_{aq}, the levels of voltage drops in the synchronous inductive resistances of the stator winding on the longitudinal and lateral axes, taken with the opposite sign, i.e.

$$E_{ad} = -jx_d I_d \tag{3.5}$$

$$E_{aq} = -jx_q I_q \tag{3.6}$$

We similarly deduce

$$E_r = -I_r \tag{3.7}$$

Adding geometrically E. M. F. E, E_{ad}, E_{aq} and E_r, we obtain the voltage of the motor $U_д$, that is

$$U_д = E + E_{ad} + E_{aq} + E_r \tag{3.8}$$

The vector of the network voltage U, which the motor is connected to, by the magnitude is equal, and by the direction is opposite to the voltage of the motor, that is

$$U = -U_д = -E - E_{ad} - E_{aq} - E_r \tag{3.9}$$

or, taking into account (3.5) - (3.7)

$$U = -E + jx_d I_d + jx_q I_q + I_r \qquad (3.10)$$

When the motor is running, the voltage vector $U_д$does not lag behind, as in the case of the generator, from the vector E. M. F. *E.*, but is ahead of it by the angle δ. Considering 0 in the generator mode as positive and in the motor mode negative, for vector projections of the motor voltage we obtain expressions identical with the formulas (2.10) for the generator [23].

2.3. The equation of the constant mode of the synchronous machine for currents and voltages

Designing the polygons of E. M. F. and those of the voltage drops of both the generator and the motor on the longitudinal and lateral axis of the rotor, it is easy to obtain the following basic equations of the steady-state mode of the nonsalient pole synchronous machine:

$$U_q = E + jx_d I_d + rI_q, \qquad (4.1)$$
$$U_d = -x_q I_q + rI_d$$

These equations can also be obtained with (62,98), putting $p = 0$ and $\Omega = 1$. Having solved these equations, we obtain for the longitudinal and lateral components of the stator current:

$$I_d = \frac{x_q(U_q - E) + rU_d}{r^2 + x_q x_d}$$

$$(4.2)$$

$$I_q = \frac{r(U_q - E) - x_d U_d}{r^2 + x_q x_d}$$

Taking into account (2.10), we rewrite the last formulas in the form:

$$I_d = \frac{x_q\,(Ucos\delta - E) + rUsin\delta}{r^2 + x_q x_d} \qquad (4.3)$$
$$I_q = \frac{r(Ucos\delta - E) - x_d Usin\delta}{r^2 + x_q x_d}$$

If we accept $r = 0$, the formulas for currents will take the following form:

$$I_d \approx \frac{Ucos\delta - E}{-\frac{U}{x_q}sin\delta} \qquad (4.4)$$
$$I_q \approx -\frac{U}{x_q}sin\delta$$

According to (33.3), between the basic units of voltage, current and resistance there is a ratio:

$$U_{Б} = E_{Б} = Z_{Б} I_{Б}, \quad (4.5)$$
$$I_{Б} = \frac{U_{Б}}{Z_{Б}} = \frac{Z_{Б} U_{Б}}{Z_{Б}^2}.$$

Dividing (4.1) into (4.5), and (4.2), (4.3) and (4.4) into (4.6), we obtain the same equations in relative and physical units. At the same time, the data in the equations in relative and physical units will have exactly the same form.

2.4. E. M. F. for lateral resistance

When solving many practical problems, the idea to introduce the notion of E. M. F. E_{xq} compatible with the lateral synchronous inductive [24] resistance or, briefly, E. M. F. in accordance with the lateral resistance appeared to be attractive. Under E. M. F. E_{xq}, they understand such a theoretical E. M. F., which should be induced by the flow of permanent magnets of the rotor in the stator winding in this steady state, as if the longitudinal synchronous inductive resistance of the machine x_d equaled its lateral synchronous inductive resistance x_q.

We'll determine the link of E. M. F. $E_{xq}$with other magnitudes of the stator winding [25].

Accordingly, we will construct a vector diagram of the synchronous generator. Let the positive direction of the d-axis pass to the left and the q-axis down from the origin of the O coordinates. Then the E. M. F. vector of the stator E, lagging from the d-axis by 90 °, will be directed upwards. Suppose from the vector *E* the vector of the generator voltage $U_г$ lags by the angle δ, and the vector of current *I* lags by the angle ψ. The longitudinal and lateral components of the current are expressed respectively through $I_d$and I_q.

We'll determine under the given conditions that value of E. M. F. of the stator winding, which would be necessary to be induced in it by the flow of permanent magnets of the rotor, if x_d equaled x_q, that is, we will determine the vector E. M. F. E_{xq}.

From the end of the generator voltage vector $U_г$, we construct in parallel with the vector of current *I* the vector of the voltage drop in the active resistance of the stator winding *rI*. The perpendicular *BC* from point *C* on the vector E. M. F. *E* under the given conditions is equal to the voltage drop $jx_d I_d$. If x_d is equal to x_q, then putting off the segment from the point *B* upwards

$$BG = jx_d I_d \quad (5.1)$$

we obtain the vector *OG* that is equal to that E. M. F., which under these conditions would have to be induced in the stator winding by the flow of permanent magnets of the rotor, or, in other words, we obtain E. M. F. E_{xq}.Thus, E. M. F. $E_{xq}$is depicted on the vector *OG* [26].

The vector of the valid E. M. F. of the stator winding *E* is determined, putting up the segment *BA* from the point *B* equal to the voltage drop$jx_d I_d$. Then

$$GA = j(x_d + x_q)I_d \tag{5.2}$$

and

$$E = E_{xq} + j\left(x_d + x_q\right)I_d, \tag{5.3}$$

where

$$E_{xq} = E - j\left(x_d + x_q\right)I_d \tag{5.4}$$

Since all the three vectors included in (5.4) are located on a single line, the equation can be written in an algebraic form as

$$E_{xq} = E + (x_d - x_q)I_d \tag{5.5}$$

Here E_{xq} and E are always positive values of the modules of corresponding vectors, and $I_d$is the longitudinal current of the stator, which has in the given diagram under the inductive load the negative value. Therefore, between the first and second members (in 5.5) there is a plus sign, but not a minus one. The hypotenuse *ΔBCG*

$$\overline{CG} = \frac{x_q I_q}{\cos\psi} = x_q I, \tag{5.6}$$

that is equal to the voltage drop across the resistance x_q from the full current *I*. We continue the hypotenuse *CG* to the intersection with the horizontal vector *AH*. The hypotenuse of obtained at that *ΔAHG* will then be equal to

$$\overline{GH} = \frac{G\bar{A}}{\sin\psi} = \frac{(x_d - x_q)I_d}{\sin\psi} = \left(x_d - x_q\right)I\,. \tag{5.7}$$

In the same Δ*AHG* the catheter

$$AH = GHcos\psi = (x_d - x_q)I_d. \tag{5.8}$$

It follows from this [26] that the vector E. M. F. according to the lateral resistance $E_{xq}$is equal to the geometric sum of the vectors: the voltage U_r, the voltage drop in the active resistance *rI*, and the voltage drop $jX_q I$ in the resistance $x_q$from the full current, that is

$$E_{xq} = U_{\text{г}} + \left(r + jx_q\right)II \tag{5.9}$$

Expressing the vectors included in the last equation by the corresponding complexes and denoting

$$r + jx_q = Z_q \tag{5.10}$$

with (5.9) we obtain the following expression for E. M. F. according to the lateral resistance

$$E_{xq} = U_{\text{г}} + Z_q \tag{5.11}$$

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To solve certain problems, the vectors jx_qI and jx_qI_q are advisable to be represented in the form of the following sum:

$$\left.\begin{aligned} jx_q\bar{I} &= jx_{aq}\bar{I} + jx_s\bar{I}\,, \\ jx_q\overline{I_q} &= jx_{aq}\overline{I_q} + jx_s\overline{I_q}. \end{aligned}\right\} \tag{5.12}$$

The vector $E_i$in this diagram, which is equal to the sum of the voltage vectors $U_{\text{г}}$, the voltage drop in the active resistance rI, and the voltage drop in the inductive diffusion resistance x_sI is that internal E. M. F., which induces in the stator winding by the resulting flow, generated by m. f. of the excitation winding F_f and m. sf. of the armature reaction F_m.

The projection of the vector of the internal E. M. F. E_i onto the lateral axis of the rotor q, denoted by E_{iq} , is that E. M. F., which induces in the stator phase by the magnetic flux produced by the algebraic sum F_{id} of m. f. of the excitation winding F_f and m. f. of the longitudinal armature reaction F_{md}. Since the saturation, to a large extent, affects the value E_{iq} , it should be taken into account when solving practical problems, using E_{iq} . The vector $x_{fq}I_q$ is equal according to the amplitude, and is opposite according to the direction to E. M. F., which induces in the stator phase by the magnetic flux, created by m. f. of the latral armature reaction F_{md} .

In nonsalient pole machines [27], the magnetic flux of the lateral armature reaction passes a significant part of the path through the air, so the resistance of the rest of the path (in the magnetic circuit) and the effect of saturation on the magnitude $x_{fq}I_q$can be neglected without making a big error. It turns out that

$$E_{xq} = E_{iq} - x_{aq}I_q \tag{5.13}$$

Here I_d is the longitudinal current of the stator. At inductive loading, the numerical value $I_q$is negative, but with capacitive load is positive.

The value $E_{iq}$is determined by the actual characteristic of no-load operation, respectively, of the algebraic sum F_{id} of M.F. of permanent magnets $F_f$and m. f. of the longitudinal armature reaction F_{md}. The last position is conventionally written in the form

$$F_{id} = F_f + F_{md} \rightarrow E_{iq}.$$

If, the current of excitation at the change of current $I_q$remains unchanged, then F_{id}, consequently, E_{iq} and E_{xq}will uniquely depend on the stator's longitudinal current I_d.

Knowing the characteristic of no-load operation, m. f. F_f and the inductive resistance of amature reaction x_{aq}, one can construct the dependence E. M. F. according to the lateral resistance $E_{xq}$from the stator's longitudinal current I_d, as recommended by the method of D. A. Gorodskyi.

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Let's construct in relative units the characteristic of no-load operation.

Voltage $U_г$ and E.M.F.E. for no-load operation are identical and equal to E. M. E_{xq}.

Let's find m. f. of the excitation winding in relative units $F_{*f} = Ob$ and any values of the longitudinal current, for example the nominal I_{*d}. Let's calculate the longitudinal n. s. of the armaure reaction $F_{*md}$with this current and put it to the left of point b in the form of segment bc. On a perpendicular from point З, we put the segment $\overline{cd}$, equal to $x_{*aq}I_{*d}$. Through points b and d we draw a direct line to its intersection with the axis of the ordinate at the point k. According to the characteristic of no-load operation, the segment ce is equal to e. m. f., which corresponds to the difference of n. s. $F_{*id} = F_{*f} - F_{*md}$,that is, the segment ce is equal to the lateral component of the internal e. m. f. E_{*iq}.

Since the load has an inductive character and the current $I_{*d}$is negative, then, according to (5.13), by expressing $ef = x_{*aq}I_{*d} = cd$, we obtain the ordinate cf, which will be equal to E. M. F. according to the lateral resistance $E_{*xq}$with the taken longitudinal current of the stator I_{*d}. Similarly, E. M. F. can be found for the lateral resistance with the same m. f. F_{*f} and for any other longitudinal current [28]. The construction of the curve $E_{*xq} = f(I_{*d}$, however, can be simplified. To do this, the corresponding ordinates of direct line kb, conforming to the construction $x_{*aq}I_{*d}$, should be added to the ordinate characteristics of no-load operation, and thus construct the kfa curve. The ordinates of this curve will be equal to E. M. F. E_{*xq} in p.u., and the abscissas counted from point b as from the origin of the coordinates, will be equal to the longitudinal current I_{*d}. The magnitude for the longitudinal current is determined from the following conditions: at point b we have $I_{*d} = 0$; the segment $bc = I_{*d}$ and segment bO are equal to such a longitudinal current, at which m. f. of the longitudinal armature reaction $F_{*md}$is equal to m. f. of the excitation winding F_{*f}. The curve $\mathrm{E_{*xq}} = \mathrm{f(I_{*d})}$ is used in the calculation of the stability of saturated nonsalient pole synchronous machines [29].

Example of e. m. f. application according to the lateral resistance

It is given: voltage [30] on the clamps of the generator $U_г$; the current strength I; the power factor $cos\ \varphi$; the active resistance of stator winding r; the unsaturated value of the longitudinal synchronous inductive resistance x_d; the lateral synchronous inductive resistance x_q.

Determine: the percentage increase in voltage at load drop $\Delta u\%$, the excitation current I_f.

Graphic solution. Let us direct the voltage vector U_r horizontally. At the angle φ to the voltage vector $U_г$, we construct the current vector I. From the end of the voltage vector D in parallel with the current vector, we construct the vector of voltage drop in the active resistance rI. Then, from point C perpendicular to rI plot the vector $j\ x_qI$ of the voltage drop in the resistance x_q. The end of the last vector will be connected with the point O. Vector OG is E. M. F. $E_{xq.}$ On the continuation of the vector E_{xq}we construct the vector GA, equal to $j\left(x_d - x_q\right)I_d$.

The OA vector will be equal to the unsaturated value of E. M. F. $E_{\text{НН}}$, induced in the stator phase by the flow of the rotor.

According to the rectified characteristic of no load operation, we determine the current of excitation $I_f = O\alpha$, corresponding to E. M. F. $E_{\text{НН}} = \alpha A$.

The actual E. M. F. E. in the stator phase with the excitation current I_f through saturation will be less than $E_{\text{Н.Н}}$ by the value of AB.

The percentage increase in voltage at load dump is found by the formula

$$\Delta u\% = \frac{E-U_r}{U_r}100\%$$

Analytical solution. Let us direct [31] the vector of voltage $U_{\text{г}}$ along the axis of real values. In this case, the complex expressions for the voltage and current will take the form

$$\mathring{U}_r = U_r$$

$$\dot{I} = I\cos\varphi - jI\sin\varphi = I_A + jI_r,$$

where I_A, I_r is the active and reactive components of the current.

The complex expression for the voltage drop in the imaginary resistance of the machine Z_q can be represented as

$$Z_q I = (r + jx_q)(I_A - jI_r) = (rI_A + x_q I_r) + j\,(x_q I_A - rI_r) = DG$$

The complex expression for E. M. F. according to the lateral resistance E_{xq} will obviously have the form of

$$\dot{E}_{xq} = (U_{\text{г}} + rI_A + x_q I_r) + j(x_q I_A - rI_r) = OG$$

The module of the vector E. M. F. according to the lateral resistance E_{xq} is equal to

$$E_{xq} = \sqrt{(U_{\text{г}} + rI_A + x_q I_r)^2 + (x_q I_A - rI_r)^2}$$

$$\cos\delta = \frac{U_{\text{г}} + rI_A + x_q I_r}{E_{xq}}$$

and

$$\psi = \varphi + \delta;$$

then the longitudinal component of the stator current is

$$I_d = I\sin\psi$$

The unsaturated value of e. m. f., induced in the stator phase by the rotor flux, is equal to

$$E_{\text{НН}} = E_{xq} = (x_d - x_d)I_d$$

Further, the solution to the problem is identical as in the graphical method.

2.5. Vector diagram of unsaturated nonsalient pole generator

In a nonsalient pole machine one can accept $x_d = x_q$and $x_{ad} = x_{aq}$. At the same time, E. M. F. according to the lateral resistance E_{xq}will be equal to E. M. F. *E*.

In a nonsalient pole machine [32], the magnetic resistance to the main flow will be identical in all directions, which makes it possible to find the value of the inner E. M. F. E_i for the proper expression of no-load operation in accordance with the geometric sum F_i of the winding m. f. for excitation F_f and m. f. of the armature reaction F_m.

The vector E_i lags in phase from the vector of m. f. F_i by 90 °. Neglecting the active resistance of the stator winding, the diagram can be presented in a simplified form. From the simplified vector diagram we will determine that E. M. F. of the stator winding

$$E = \sqrt{{U_\mathrm{г}}^2 \cos^2 \varphi + (U_\mathrm{г} sin\varphi + X_d I)^2} \tag{6.1}$$

If the terminal voltage and the generator current are equal to their nominal value, that is, if $U_\mathrm{г} = U_\mathrm{Б}$ and $I = I_\mathrm{Б}$ then, dividing both parts (6.1) into $U_\mathrm{Б}$, we obtain for E. M. F. of stator windings in p. u. such an expression:

$$E_* = \sqrt{\cos^2 \varphi + (\sin \varphi + x_{*d})^2} \tag{6.2}$$

where

$$x_{*d} = \frac{x_d I_\mathrm{Б}}{U_\mathrm{Б}} \tag{6.3}$$

is a synchronous inductive resistance on the longitudinal axis in p. u.

2.6. Vector diagram of a saturated nonsalient pole generator

In saturated nonsalient pole synchronous machines after the summation of m. f. of the excitation winding F_f and m. f. of the armature reaction $F_m$according to the resultant m. f. F_i they find the saturated value of the internal E. M. F. E_i, using the real no-load characteristic. We denote the relation of the unsaturated value of the internal E. M. F. $E_{\mathrm{інн}}$ to the saturated by its value E_i the coefficient α_μ, that is

$$\frac{E_{\mathrm{інн}}}{\mathrm{E_i}} = \frac{MD}{ND} = \alpha_\mu$$

The coefficient $\alpha_\mu$is greater than unity and depends on the value of m. f. F_i , that is, from the position of point *D*. With increasing the saturation α_μ, increases [33].

The inductive resistance of the armature reaction on the longitudinal axis, taking into account the saturation $x_{ad\mu}$, will obviously be less than the unsaturated one, the value of the same magnitude by α_μ times, i.e. $x_{ad\mu} = \frac{x_{ad}}{\alpha_\mu}$.

After these preliminary remarks, we turn to the construction of a vector diagram of a saturated nonsalient pole synchronous generator.

Vector m. f. of the excitation winding F_f is constructed on the d (7.1) axis in the positive direction. At the angle of $90° + \psi$ to it, in the direction of the lag, we plot a vector of current *I*. From the end of F_f in parallel to *I* we construct the vector of m. f. of the armature reaction F_m. The closing vector will be equal to the resultant m. f. F_i, the active **pension system of the machine**. M. F. F_i create a magnetic flux Φ_i, which induces in the phases of the stator internal E. M. F. E_i. Knowing F_i we will find the saturated E_i and unsaturated $E_{\text{iНН}}$ values of the internal E. M. F. and, consequently, the points D and B, respectively, according to the actual and straightforward no-load operation characteristics[34].

From the end of the vector $E_{\text{iНН}}$, perpendicular to the current, we construct the vector $jx_{ad}I$, where x_{ad} is the unsaturated value of the longitudinal inductive resistance of the armature reaction. The closing vector $E_{\text{НН}}$ is the unsaturated value of e. m. f. in the stator winding.

To construct the voltage vector $U_\text{г}$ from the end of the vector E_i, we put the vector $DG = jx_sI$ perpendicularly the current downwards, and from the beginning of the last vector we will construct the vector r*I* parallel to the current. The closing vector U_r will be the terminal voltage of generator. If we plot the vector $j\frac{x_{ad}}{\alpha_\mu}I = DC$ from the end of the vector E_i perpendicular to the current, then we find the vector $\frac{E_{\text{НН}}}{\alpha_\mu} = OC$ that is equal to that value of E. M. F., which would be induced in the winding of no-load stator, as if the saturation at no-load were the same as at loading, that is would correspond to point *N* of the no-load characteristic.

If M. F. of the excitation F_f will be greater than M. F. F_i, which takes place during the non-operating run, then the coefficient α_μ will be greater than at loading, and therefore the true value of E. M. F. E at load dump will be less than *OC* [35].

2.7. Active power of the nonsalient pole synchronous machine at the assumption that $r = 0$

Neglecting the active resistance of the stator winding, let's assume the vector diagram of the synchronous generator.

The power of the synchronous generator, provided to the network, is equal to $P = 3UI\cos\varphi$, where $U = U_\text{г}$, or according to

$$P = 3UI\cos(\psi - \delta) = 3U(I\cos\psi\, cos\delta + I\sin sin\delta).$$

Since, in addition, according to

$$x_qI_q = Usin\delta,$$

$$x_d I_d = E - U cos\delta,$$

then

$$P = 3U(\frac{U}{x_q} sin\delta cos\delta + \frac{E}{x_d} sin\delta - \frac{U}{x_d} sin\delta cos\delta) \quad (8.1)$$

or

$$P = \frac{3UE}{x_d} \sin\delta + \left(\frac{1}{x_q} - \frac{1}{x_d}\right)\frac{3U^2}{2} sin2\delta \quad (8.2)$$

The expression for power could also be obtained proceeding from the vector diagram of the motor. The first component of the power in the natural expression (8.2) is proportional to *E*, and hence the rotor's flux. The second component of power is due to a difference in the values of inductive stator resistances on the longitudinal and lateral axes of the machine.

In nonsalient pole machines $x_d = x_q$, hence, the second component of the power in (8.2) is equal to zero. The moment in watt-radians is numerically equal to the power and therefore expressed by the same formula (8.2). Expression (8.2) for the moment of rotation and power in the steady state is equal to the previously obtained in a different mode expression (44, 60) [36]. The expression for power and moment in p. u. can be obtained by dividing (8.2) into the baseline power and the moment

$$P_Б = M_Б = \frac{1}{X_Б} 3\left(\frac{U_Б}{\sqrt{2}}\right)^2 = \frac{3}{X_Б} U_{эБ}^2, \quad (8.3)$$

where $U_{эБ}$ is the basic unit for effective values of voltage and E. M. F., if at that $U = \frac{U_Б}{\sqrt{2}}$, then the power and moment in p. u. are equal to

$$P_* = M_* = \frac{E_*}{x_{*d}} sin\delta + \frac{1}{2}\left(\frac{1}{x_{*q}} - \frac{1}{x_{*d}}\right) sin2\delta \quad (8.4)$$

2.8. Reactive power of an unsaturated nonsalient pole synchronous generator at R = O

The vector diagram of an unsaturated nonsalient pole synchronous generator while neglecting the active resistance of the stator winding has the form shown in 6.2.

According to the rectangular ΔOAB at $U_г = U$we have

$$E^2 = U^2 \cos^2\varphi + (U\sin\varphi + x_d I)^2$$

or

$$E^2 = U^2 + x_d^2 I^2 + 2x_d UI\sin\varphi.$$

Let us express the reactive power in terms of *Q*, i.e.

$$Q = 3UIsin\varphi,$$

then

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$$E^2 = U^2 + x_d^2 I^2 + \frac{2}{3} x_d Q. \tag{9.1}$$

Let us express the reactive power through the voltage U, E. M. F. E and the angle δ. For this purpose, from the point C, we drop the perpendicular CD onto the vector E. M. F. E.

With ΔADC and ΔODC we have:

$$x_d I cos\psi = U \sin\delta,$$
$$x_d I \ sin\ \psi = E - U\ cos\delta.$$

We reduce to the square and sum up the last two equalities

$$x_d^2 I^2 = U^2 + E^2 - 2EU \cos\delta. \tag{9.2}$$

Substituting (9.2) into (9.1), we obtain

$$Q = 3\frac{EU}{x_d}\cos\delta - 3\frac{U^2}{x_d}. \tag{9.3}$$

Dividing the last expression into the base power (8.3), we obtain for reactive power in p. u. the expression

$$Q_* = \frac{E_* U_*}{x_{*d}}\cos\delta - \frac{U_*^2}{x_{*d}^2}. \tag{9.4}$$

For the similar nonsalient pole machine, the active power in physical and relative units will be in accordance with (8.2) and (8.3) and is equal to

$$P = \frac{3EU}{x_d}\sin\delta, \tag{9.5}$$

or

$$P_* = \frac{E_* U_*}{x_{*d}}\sin\delta\ . \tag{9.6}$$

2.9. Active and reactive power in accordance to the dissipation resistance

The power of dissipation resistance is understood to be the power of a machine that it would develop if the inductive resistance [37] of dissipation and the active resistance of the stator winding were equal to zero, and the saturation coefficient at no-load was equal to the saturation coefficient under load. Given these conditions, obviously, the following equalities (7.1) will take place:

$$\begin{gathered} U_{\text{г}} = E_i \\ E = \frac{E_{\text{нн}}}{\alpha_\mu} \\ \delta = \delta_{\text{i}} \\ x_d = x_{ad} \end{gathered} \tag{10.1}$$

The saturated value of the synchronous longitudinal inductive resistance in this case will be equal to $\frac{x_{ad}}{\alpha_\mu}$

Substituting the corresponding values from (10.1) into (9.5) and (9.3), we obtain the following expressions for active and reactive powers [39] according to the dissipation resistance:

$$P_i = \frac{3E_{\text{НН}}E_i}{x_{ad}}\sin\delta_{\text{i}} \tag{10.2}$$

and

$$Q_i = \frac{3E_{\text{НН}}E_i}{x_{ad}}\cos\delta_i = \frac{3E_i^2\alpha_\mu}{x_{ad}} \tag{10.3}$$

in p. u. will also have the following form:

$$P_{*i} = \frac{E_{*\text{НН}}E_{*i}}{x_{*ad}}\sin\delta_i \tag{10.4}$$

$$Q_{*i} = \frac{E_{*\text{НН}}E_{*i}}{x_{*ad}}\cos\delta_i - \frac{E_{*i}^2\alpha_\mu}{x_{*ad}} \tag{10.5}$$

where x_{*ad} are the unsaturated values of the inductive resistance of the armature reaction in p. u. The reactive power on the clamps of the machine Qwill be less than the power Q_{i} by the unity of the reactive power of dissipation $3x_sI^2$, that is

$$Q = Q_i - 3x_sI^2 \tag{10.6}$$

and the power of the machine according to the dissipation resistance seems to be equal to

$$S_i = 3E_iI = \sqrt{P_i^2 + Q_i^2} \tag{10.7}$$

and $$I^2 = \frac{P_i^2+Q_i^2}{(3E_i)^2},$$

$$Q = Q_i - \frac{P_i^2+Q_i^2}{E_{*i}^2}x_{*s}. \tag{10.9}$$

If we neglect the loss of active power in the active resistance of the stator winding, we can equate the active power on the clamps of the machine P of the active power according to the dissipation resistance P_i , i.e. it is possible to express

$$P = P_i$$

2.10. Experimental determination of parameters of steady-state modes of the synchronous machine

The value of parameters of synchronous machines is appreciably dependent on the methods of their determination. Here is a brief description of the measurement methods that give the value of parameters that are suitable for use in the equations of most transient processes of the synchronous machine.

1. The active resistance of the phases of the stator r and the excitation winding r_f can be measured at a constant current by the method of ammeter and voltmeter.

2. The resistance of the excitation winding Z_f of the machine without a damper winding can be measured by the method of ammeter and voltmeter at alternating current of nominal frequency [40]. The winding of the stator in this case should be opened, and the rotor is stationary. The fully inductive resistance of the excitation winding is calculated by the following formula

$$x_f = \sqrt{Z_f^2 - r_f^2}. \tag{11.1}$$

In most cases, without making a big error, one can equate the inductive resistance x_f of the full resistance Z_f, that is, to assume $x_f \approx Z_f$.

3. To determine the resistance of the mutual induction of the stator winding with the winding of excitation x_{af}, the following method can be used. With an open winding of the stator and the rotor spinning with the synchronous speed, the constant current I_f passes through the excitation winding, and the phase voltage U is measured.

Then

$$x_{af} = \frac{E_m}{I_f} = \frac{\sqrt{2}U}{I_f}. \tag{11.2}$$

4. The resistance of mutual induction of the excitation winding with the stator winding on the longitudinal axis $x_{fa}$of the machine without a damper winding can be defined as follows.

With an open winding of excitation and a stationary rotor a constant three-phase current of direct sequence passes through a stator winding [41]. Further, the effective values of the current in the stator I and the voltage on the clamps of the excitation winding U_f are measured.

Then

$$x_{fa} = \frac{U_f}{I} \tag{11.3}$$

5. Synchronous inductive resistances on the longitudinal and lateral axes x_d and x_q are very simply and accurately determined by the method of sliding.

The stator winding with a closed excitation winding is supplied with a reduced voltage of normal frequency fed from an independent current source. The rotor is spinning at a speed close to the synchronous one and then the excitation winding of the machine is opened. In a nonsalient pole machine, if there is a slip, the magnetic resistance to the flow generated by m. f. of the stator will change, while the inductance and current of the stator change accordingly [42].

Since the external voltage source has a finite power, then the voltage applied to the winding of the stator does not remain constant, but fluctuates with the stator flux, and also the minimum voltage on the clamps U_{min} corresponds to the maximum current in the stator I_{max}, and vice versa. The maximum of the stator current corresponds to the coping of the stator flux with the lateral axis of the rotor, and the minimum of the stator current corresponds to the coincidence of the stator flux with the longitudinal axis of the rotor. Therefore, the inductive resistance should be calculated according to the formulas:

$$x_d = \frac{U_{max}}{\sqrt{3I_{min}}}, \tag{11.4}$$

$$x_q = \frac{U_{min}}{\sqrt{3I_{max}}}.$$

The highest U_{max}, I_{max} and the smallest U_{min}, I_{min} the effective values of the voltage and current can be directly calculated by means of devices or determined by oscillograms.

2.11. Equation for the stationary rotor current

Example 1. Currents passing through the stator winding of a synchronous machine without a damper winding, the data of which are given in Table. 111,1:

$$\begin{aligned} i_a &= 0{,}98 \cdot \sqrt{2 \sin t}, \\ i_b &= 10{,}65 \cdot \sqrt{2 \sin(t - 93°)} \\ i_c &= 10{,}65 \cdot \sqrt{2 \sin(t + 93°)} \end{aligned} \tag{12.1}$$

The excitation winding is short-circuited. The rotor is stationary. The longitudinal axis of the rotor composes with the axis of phase *a* the angle $\theta_{ad} = 90°$ (12.1).

The equation for the current of the rotor $i_f$in a steady state [43] is to be composed.

Solution. For a machine without a damper winding with a short-circuit excitation winding (U_f= 0) equations (15.21) and (14.42) take the following form:

$$i_f + \frac{d\psi f}{dt} = 0 \tag{12.2}$$

$$\psi_f = x_f i_f + x_{fa} i_d \tag{12.5}$$

replacing in (12.2) the derivative by the expression from (12.3), we obtain

$$ri_f + \frac{d}{dt}(x_f i_f + x_{fa} i_d) = 0 \tag{12.4}$$

To determine the direct-axis component, constructing the vectors of the stator current, we use the formula (18, 6), which in axes *d*, *q* has the following form

$$i_d = \frac{2}{3}[i_a \cos\theta_{ad} + i_b cos[\theta_{ad} - 120°] + i_c cos(\theta_{ad} + 120°)]. \tag{12.5}$$

Substituting in (12.5) the expressions for currents from (12.1) at θ_{ad}= 90 °, we obtain

$$i_d = \frac{2}{3}\left[10{,}65 \cdot \sqrt{2sin(t-93°)}\cos(-30°) + 10{,}65 \cdot \sqrt{2}\sin(t+93°)\,cos210°\right] =$$
$$= -12{,}3 \cdot \sqrt{2}\cos t$$

Substituting the last expression in (12.4) we obtain the following differential equation:

$$x_f \frac{di_f}{dt} + r_f i_f + 12{,}3 \cdot \sqrt{2x_{fa}}\sin t = 0 \qquad (12.6)$$

Assuming that the initial conditions are equal to zero, that is, assuming that with $t = 0$ also i_f= 0, we write (12.6) in the operational form:

$$\mathrm{px_f i_f(p) + r_f i_f(p) + 12{,}3 \cdot \sqrt{2x_{fa}}\frac{p}{1+p^2}};$$

from here

$$\mathrm{i_f(p) = -12{,}3 \cdot \sqrt{2x_{fa}}\frac{p}{(r_f+px_f)(1+p^2)}}. \qquad (12.7)$$

Let us expand the resulting operational expression initially approximated, putting r_f = 0. Then

$$\mathrm{i_f(p) = -12{,}3 \cdot \sqrt{2}\frac{x_{fa}}{x(1+p^2)}},$$

but the origin of this operating expression has the form

$$i_f = 12{,}3 \cdot \sqrt{2}\frac{x_{fa}}{x_f}\cos t - 12{,}3\sqrt{2}\frac{x_{fa}}{x_f}$$

Consequently, in case of $r_f = 0$ and a «sudden» passage of currents (12.1) through the stator winding, the excitation current will consist of two components, namely the periodic and constant ones [44].

If there is an active resistance excitation in the winding, the constant component will be attenuated and in the steady state the current of excitation will be equal to

$$i_f = 12{,}3 \cdot \sqrt{2}\frac{x_{fa}}{x_f}\cos t = 12{,}3 \cdot \sqrt{2}\frac{9{,}7}{3{,}79}\cos t = 31{,}5 \cdot \sqrt{2}\cos t \qquad (12.8)$$

To take into account the effect of the active resistance r_f on the magnitude of the excitation current, it is necessary to expand the operator expression (12.7). Let us use the expansion theorem for this purpose.

Let's put:

$$\mathrm{M(p) = p},$$

$$\mathrm{N(p) = (r_f + px_f)(1+p^2)},$$

$$\mathrm{C = -12{,}3 \cdot \sqrt{2}x_{fa}}.$$

here

$$\mathrm{N'(p) = 3x_f p^2 + 2pr_f + x_f}.$$

The roots of the equation $N(\mathrm{p}) = 0$ will be:

$$\mathrm{p}_1 = -\frac{\mathrm{r_f}}{\mathrm{x_f}} = -\frac{1}{\mathrm{T_{do}}}, \qquad \mathrm{p}_2 = \sqrt{-1} = \mathrm{j}, \quad \mathrm{p}_3 = -\mathrm{j}.$$

On the basis of the expansion formula [see (26), annex 6] we will put:

$$\mathrm{i_f} = \mathrm{C}\left[\frac{\mathrm{x_f}}{\mathrm{r_f^2} + \mathrm{x_f^2}} \mathrm{e}^{-\frac{\mathrm{t}}{\mathrm{T_{do}}}} + \frac{\mathrm{e^{jt}}}{2(\mathrm{jr_f} - \mathrm{x_f})} - \frac{\mathrm{e^{-jt}}}{2(\mathrm{jr_f} + \mathrm{x_f})}\right]$$

or

$$i_f = \frac{Cx_f}{r_f^2+x_f^2} e^{-\frac{t}{T_{do}}} + \frac{Cr_f}{r_f^2+x_f^2}\sin t - \frac{Cx_f}{r_f^2+x_f^2}\cos t\,. \qquad (12.9)$$

By trigonometric transformations, one can also deduce for i_f the expression

$$i_f = \frac{Cx_f}{r_f^2 + x_f^2} e^{-\frac{t}{T_{do}}} - \frac{C}{\sqrt{r_f^2 + x_f^2}}\cos(t + \varphi_f),$$

where

$$\mathrm{tg}\varphi_\mathrm{f} = \frac{\mathrm{r_f}}{\mathrm{x_f}}$$

In a steady state, the first decaying part in equations for i_f will be equal to zero. On the basis of (12.9) and data presented in tables 111.1 for the excitation current in a steady state we will write

$$i_f = -\frac{12{,}3 \cdot \sqrt{2} x_{fa} r_f}{r_f^2 + x_f^2}\sin t + \frac{12{,}3 \cdot \sqrt{2} x_{fa} r_f}{r_f^2 + x_f^2}\cos t,$$

or

$$i_f = -1{,}127 \cdot \sqrt{2}\sin t + 31{,}45 \cdot \sqrt{2}\cos t \qquad (12.10)$$

The equation for the excitation current, obtained experimentally (oscillogram), has the following form

$$i_f = 30{,}2 \cdot \sqrt{2}\cos t \qquad (12.11)$$

From the comparison of equations (12.8) and (12.10) with each other and with equation (12.11), it follows that neglecting the active resistance has little effect on the magnitude of the steady value of the excitation current.

The estimated value of the current in the rotor winding, calculated at $r_f = 0$, differs from its experimental value by approximately 4.3%.

Example 2. The conditions of this example differ from the conditions of the previous one by the fact that the longitudinal axis of the rotor coincides with the axis of the phase a, that is, the angle $\theta_{ad} = 0$ (12.2), and in the stator phases the following currents flow:

$$i_a = 11{,}3 \cdot \sqrt{2 \sin \mathrm{t}},$$
$$i_b = 5{,}65 \cdot \sqrt{2 \sin (\mathrm{t} - 180°)}\ , \qquad (12.12)$$
$$i_c = 5{,}65 \cdot \sqrt{2 \sin(\mathrm{t} + 180°)}.$$

It is necessary to compose an equation for the current of the rotor i_f in a steady state.

Solution. Substituting in (12.5) the expression for currents from (12.12) and $\theta_{ad} = 0°$, we obtain for the longitudinal component the representation of the stator current vector

$$i_d = 11{,}3\sqrt{2}\sin t.$$

Substitution of the obtained expression for i_d in (12.4) results in a differential equation

$$x_f \frac{di_f}{dt} + r_f i_f + 11{,}3 \cdot \sqrt{2} x_{fa} \cos t = 0.$$

Transforming this equation into an operating form, we obtain for the excitation current

$$\mathrm{i_f(p)} = -11{,}3 \cdot \sqrt{2}\mathrm{x_{fa}} \frac{\mathrm{p}^2}{(\mathrm{r_f} + \mathrm{px_f})(1 + \mathrm{p}^2)}$$

As shown by the solution of the preceding example, the active resistance of the excitation winding can be neglected. Then, based on the last equation we obtain

$$\mathrm{i_f(p)} = -11{,}3 \cdot \sqrt{2} \frac{x_{fa}}{x_f} \frac{p}{1 + p^2}.$$

Taking advantage of (12.5), and the data of the table 111, 1, we obtain for the rotor current the equation of the form

$$\mathrm{i_f} = -11{,}3 \cdot \sqrt{2} \frac{x_{fa}}{x_f} \sin t = -11{,}3\sqrt{2} \frac{9{,}7}{3{,}79} \sin t = -28{,}9 \cdot \sqrt{2} \sin t$$

The equation of the curve of current in the rotor winding, obtained for the conditions of this experiment from the oscillogram $i_f = -27{,}6 \cdot \sqrt{2}\sin t$. The estimated value of the current at $r_f = 0$ differs from the experimental one by 4.7%.

2.12. Equation for phase voltages and currents

Example 3. The machine parameters are shown in the table 111.1. The stator winding is open. The rotor is stationary and its longitudinal axis coincides with the axis of phase *a* (12.2). The current of 60 Hz, which varies according to the equation given below, passes through the excitation coil.

$$i_f = 7{,}65 \cdot \sqrt{2} \sin t \qquad (13.1)$$

The equation of the voltage curve U_a on the clamps of the phase *a* in a steady state is to be derived.

Solution. According to the conditions, we have:

$$i_d = i_d(p) = i_q = i_q(p) = i_0 = 0;$$
$$\theta_{ad} = 0; \quad \vartheta_r = 0. \tag{13.2}$$

In addition, according to (13.1), (13.2), (14, 44) and (15, 19):

$$I_f(p) = 7{,}65 \cdot \sqrt{2}\frac{p}{1+p^2}; \tag{13.3}$$

$$\psi_0 = x_0 i_0 = 0; \tag{13.4}$$
$$u_0 = ri_0 + \frac{d_{\psi 0}}{dt} = 0.$$

The steady state value of the stator voltage does not depend on the initial value of the excitation current, therefore, according to (14, 42), it is possible to accept:

$$i_f(0) = 0,$$
$$\psi_d(0) = x_{af} i_f(0) + x_d i_d(0) = 0, \tag{13.5}$$
$$\psi_q(0) = x_q i_q(0)$$

having in mind (13.2) - (13.5), according to (62.96) we obtain:

$$u_p(p) = u_p = 0, \tag{13.6}$$
$$u_p(p) = u_p = 0, \tag{13.7}$$

using the formula (13.5), we find the origin of the last expression

$$u_d = 7{,}65 \cdot \sqrt{2} x_{af} \cos t = 7{,}65 \cdot \sqrt{2} \cdot 6{,}43 \cos t = 49{,}2 \cdot \sqrt{2} \cos t \tag{13.8}$$

in relation to the voltage of the stator phase *a* and the axes *d, q* the formula (18.7) has the following form

$$u_a = u_d \cos\theta_{ad} - u_q \sin\theta_{ad} + u_{0.} \tag{13.9}$$

In conditions of this problem, the instantaneous value of the phase voltage *a* will be equal to

$$u_a = u_d = 49{,}2 \cdot \sqrt{2} \cdot \cos t. \tag{13.10}$$

Equation derived experimentally

$$u_a = 47 \cdot \sqrt{2} \cos t. \tag{13.11}$$

The difference in the experimental and calculated values of the voltage is 4.5%

In this example, the equations in the operator form do not give an advantage. Using the equations in the classical form, we would solve the problem more easily. In fact, bearing in mind (13.1) and (13.2), with (15, 15), (14. 42) and (15. 19), we obtain:

$$u_d = \frac{d_{\psi d}}{dt}; \quad u_q = 0, \tag{13.12}$$

$$u_0 = 0; \tag{13.13}$$

$$\psi_d = x_{af} i_f \tag{13.14}$$

from (13.12) and (13.14) it follows

$$u_d = x_{af} \frac{d_{if}}{dt}. \tag{13.15}$$

The substitution of value i_f from (13.1) into this equation gives

$$u_d = 7{,}65 \cdot \sqrt{2} x_{af} \cos t = 49{,}2 \cdot \sqrt{2} \cos t \tag{13.16}$$

Substituting the voltage components from (13.12), (13.13) and (13.16) into (13.9), we obtain

$$u_a = 49{,}2 \cdot \sqrt{2} \cos t \tag{13.17}$$

In this case, only the transformer E. M. F. induces in the stator winding.

Example 4. The machine parameters are shown in the table 111.1. The rotor rotates at a constant synchronous speed. The stator winding is open. The current passes through the excitation winding

$$i_f = 7{,}7 * \sqrt{2} \sin t \tag{13.18}$$

The equation of the curve of voltage variation of phase *a* is to be derived.

Solution. Under the given conditions we have:

$$i_d = i_q = i_0 = 0; \ \ \omega_{ad} = \omega_r = 1. \tag{13.19}$$

In accordance with (14, 42) and (14, 44):

$$\psi_d = x_{af} i_f, \ \ \psi_q = 0, \ \ \psi_0 = 0 \tag{13.20}$$

Taking advantage of the last equations, we obtain from (15, 15) and (15. 19):

$$\begin{gathered} u_d = \frac{d_{\psi d}}{dt}, \\ u_q = \psi_d, \\ u_0 = 0 \end{gathered} \tag{13.21}$$

from (13.20) and (13.21) we have:

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$$u_d = x_{af}\frac{d_{if}}{dt},$$
$$u_q = x_{af}i_f \qquad (13.22)$$

taking into account (13.18), we rewrite (13.22) in the form of:

$$u_d = 7{,}7 * \sqrt{2}x_{af}\cos t,$$
$$u_q = 7{,}7 * \sqrt{2}x_{af}\sin t. \qquad (13.23)$$

The rotor spinning takes place according to the equation

$$\theta_{\mathrm{ad}} = \theta_{\mathrm{r}} = \mathrm{t} + \theta_{\mathrm{r0}}.$$

If the initial angle $\theta_{r0}$is equal to zero, then $\theta_{ad} = t$ and the formula (13.9) for this case will take the following form

$$u_a = 7{,}7 * \sqrt{2}x_{af}(\cos^2 t - \sin^2 t) = 49{,}5 * \sqrt{2}\cos 2t. \qquad (13.24)$$

The equation of the voltage curve of phase a, obtained from the oscillogram,

$$u_a = 47{,}5 * \sqrt{2}\cos 2t. \qquad (13.25)$$

The experimental difference with the calculation performed is 4.2%.

Example 5. The rotor of the synchronous machine spins with a constant synchronous speed. The stator winding phases are short-circuited. An alternating current passes through the winding of the rotor

$$i_f = 26{,}5 \cdot \sqrt{2}\sin t \qquad (13.26)$$

The equation of current in phase a and in a steady state, neglecting the active resistance of the stator r winding is to be derived.

Solution. In accordance with the condition, we have:

$$r = 0; \quad \alpha_d = \frac{r}{x_d} = 0; \quad \alpha_q = \frac{r}{x_q} = 0; \qquad (13.27)$$
$$\dot{\theta}_{ad} = \dot{\theta}_r = 1; \quad i_0 = 0; \quad u_d = u_q = u_0 = 0.$$

To implement the solution, we use the equation (62,106). Since the value of the stator current does not depend on the initial conditions, we consider the latter to be equal to zero, i.e.

$$i_f(0) = 0; \quad \psi_d(0) = \psi_q(0) = 0. \qquad (13.28)$$

Substituting (13.27) and (13.28) into (62.106) we obtain:

$$i_d(p) = -K_d i_f(p), \quad i_q(p) = 0.$$

Or

$$i_d = -K_d i_f, \quad i_q = 0 \tag{13.29}$$

according to (62,17) and (62,75) at $\alpha_d = \alpha_q = 0$u and $\omega_r = 1$

$$\Delta_d = 1 + p^2;$$

$$K_d = \frac{x_{af}}{x_d}; \tag{13.30}$$

then

$$i_d = -\frac{x_{af}}{x_d} i_f = -26{,}5 \cdot \sqrt{2}\frac{x_{af}}{x_d} \sin t. \tag{13.31}$$

If the rotor spins according to the equation $\theta_{ad} = t$, then for the problem at hand the equation (18.7) for the instantaneous value of the current in phase *a* will take the form of

$$i_a = -26{,}5 \cdot \sqrt{2}\frac{x_{af}}{x_d} \sin t \cos t = -4{,}8\,\sqrt{2}\,\sin 2t.$$

The equation of the current curve in phase a, obtained from the oscillogram, has the form of

$$i_a = -5 \cdot \sqrt{2} \sin 2t.$$

The difference of the experiment with the calculation performed is 4%.

2.13. Equation for the voltage on the excitation winding when negative-sequence currents pass through the stator winding

Example 6. The machine parameters are shown in the table 111.1. The rotor spins at a constant synchronous speed of $\omega_r = 1$. The excitation winding is open. Symmetrical three-phase negative-sequence currents pass through the stator winding:

$$i_{a2} = i_{m2} \sin t$$
$$i_{b2} = i_{m2} \sin(t + 120°), \tag{14.1}$$
$$i_{c2} = i_{m2} \sin(t - 120°),$$

where

$$i_{m2} = 3{,}95 \cdot \sqrt{2}.$$

The equation for voltage on the clamps of the excitation winding in a steady state is to be derived.

Solution. Under the terms of this example $i_f = 0$.

Then, for the voltage and flux linkage of the excitation winding in accordance with (15.21) and (14.43), we can write the following equation [44]:

$$u_f = \frac{d_{\psi f}}{dt}, \ \ \psi_f = x_{fa} i_{d2},$$

from which

$$u_f = x_{fa} \frac{di_{d2}}{dt}, \tag{14.2}$$

where i_{d2} is the longitudinal component of the stator negative-sequence current.

Considering $\theta_{ax} = \theta_{ad} = t$ and substituting the equations (18.6) for the currents from (14.1), we obtain

$$i_{d2} = \frac{2}{3} i_{m2}[\sin t \cos t \ + sin(t + 120°) \cos(t - 120°) + \sin(t - 120°) \cos\,(t + 120°)]$$

using formula (8), we find

$$i_{d2} = i_{m2} \sin 2t.$$

Substitution of (14.3) into (14.2) gives

$$u_f = 2x_{fa} i_{m2} \cos 2t = 2 * 9{,}7 * 3{,}97 * \sqrt{2} \cos 2t = 76{,}6 * \sqrt{2} \cos 2t.$$

The equation for voltage on the clamps of the excitation winding, obtained from the oscillogram, has the form of

$$u_f = 77{,}5 * \sqrt{2} \cos 2t.$$

The difference of the experiment with the calculation performed is 1.2%.

2.14. Determination of transient and super-transient resistance

In (62.18) there is operator p, hence, the longitudinal inductive resistance of the stator winding in the transition modes is a certain function of time.

In the theory of the operating calculus, it is proved that the original value can be obtained at the initial moment by putting in the image $p = \infty$. Thus, the formula for the transient resistance of the stator winding on the longitudinal axis can be obtained from (62.18), dividing the numerator and denominator of the second term into p and putting $p = \infty$, then

$$x_d{}' = x_d - \frac{x_{af} x_{fa}}{x_f}. \tag{15.1}$$

The resulting formula is absolutely identical with the previously derived one by other means (28.3). Formulas for super-transient resistances of a synchronous machine with a damper winding will be deduced from (66.15) and (66. 30), putting into them p = ∞.

Then

$$x_d{}'' = x_d - \frac{x_{fm} x_{1d} - 2x_{1md} x_{fm} + x_f x_{1md}}{x_f x_{1d} - x_{fm} x_{1md}} x_{ad}, \tag{15.2}$$

$$x_d'' = x_q - \frac{x_{1aq}x_{a1q}}{x_{1q}}. \qquad (15.3)$$

The formulas obtained for super-transient resistances on the longitudinal and lateral axes for the ideal machine are completely identical with the previously derived by other means formulas (29.3) and (29.4).

2.15. Buildup of three-phase short-circuit current of a synchronous machine without a damper winding

Let's determine the equation of the current buildup curve in the stator winding, the clamps of which are short-circuited, if the rotor of the machine spins at a constant speed ω_r, and a constant in magnitude voltage U_f is suddenly applied to the clamping of the excitation winding.

Since the winding of the stator is short-circuited, then the longitudinal and lateral components of the voltage u_d and u_q are equal to zero. One can neglect the active resistance of the stator winding and assume the short-circuit current to be the reactive current, that is to put

$$i_d = -i_{m\kappa},\ i_q = 0 \qquad (16.1)$$

where $i_{m\kappa}$ is the amplitude of short-circuit current

Since $i_q = 0$ and there is no excitation winding on the lateral axis, then the cross flux and the lateral flux linkage are also equal to zero. Here, equation (15. 15) will have the following form:

$$\frac{d_{\psi d}}{dt} = 0,\ \ \omega_{r\psi d} = 0 \qquad (16.2)$$

since $\omega_r \neq 0$, then

$$\psi_d = 0 \qquad (16.3)$$

in addition, the equation (62.90) takes the following form

$$0 = G_{cd}U_f + x_d(p)i_d(p). \qquad (16.4)$$

From (16.4), (62.35), (62.18) and (14.31)

$$i_d(p) = -\frac{x_{af}U_f}{r_f}\frac{1}{x_d + px_d'T_{d0}} = -E_m\frac{1}{x_d + px_d'T_{d0}}. \qquad (16.5)$$

Expanding the last operator expression, we obtain for the amplitude of short-circuit current the expression

$$i_{m\kappa} = -i_d = I_{m\kappa}\left(1 - e^{-\frac{t}{T_d'}}\right), \qquad (16.6)$$

where $I_{m\kappa} = \frac{E_m}{x_d}$ is the shot-circuit current final value, and

$T_d{}' = \frac{x_d{}'}{x_d} T_{d0}$ is the constant time of the excitation winding with the short-circuited winding of the stator.

The minus sign in the formula (16.6) indicates that the shot-circuit current is directed toward the negative values of the *d* axis and, consequently, to the fact that m. f. of the armature reaction in event of short circuit has a direction opposite to m. f. of the excitation winding.

It also follows from the formula obtained that the shot-circuit current at sudden application of the excitation voltage increases in an ideal machine under the exponential law with a constant excitation winding time in case of a closed-coil stator winding $\mathrm{T_d}$, equal to the time constant of the short-circuit current transient component.

Let's write the equation for the excitation current. If the active resistance of the stator winding is assumed to be zero, then according to (16.3) and (14.42)

$$\psi_d = x_{af} i_f + x_d i_d = 0.$$

Hence from (16.6):

$$i_f = -\frac{x_d}{x_{af}} i_d = I_{m\text{к}} \frac{x_d}{x_{af}} \left(1 - e^{-\frac{t}{t_d{}'}}\right),$$

or

$$i_f = I_f \left(1 - \mathrm{e}^{-\frac{\mathrm{t}}{\mathrm{t_d}'}}\right),$$

where I_f is the constant value of the excitation current.

2.16. Increase in current of a three-phase short circuit of synchronous machine with a damper winding

Repeating the considerations given in §16 for the shot-circuit current of the machine with a damper winding at a sudden application of the excitation voltage one can be obtain the operator equation (16.4), in which the expression for $x_{\mathrm{d}}(p)$ and G_{cd} will have a more complex form of formulas [44] (16. 17) and (66.18). In this case, instead of (16.5) we obtain:

$$i_{m\text{к}}(p) = E_m \frac{1+pc}{ap^2+bp+x_d}, \tag{17.1}$$

where

$$a = x''_d T_{d0} T''_{d0},$$

$$b = x_d(T_{d0} + T_{1d0}) - x_{ad}(T_{fm} + T_{1md}), \tag{17.2}$$

$$c = T_{1d0} - T_{1md}.$$

The roots of the denominator of equation (17.1) are equal to

$$p_{1,2} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{x_d}{a}}. \tag{17.3}$$

The amplitude of the short-circuit current according to the formula (26) given in the appendix 6, can be expressed by the equation

$$i_{m\text{к}} = E_m \left[\frac{1}{x_d} + \frac{(1+p_1 c)e^{p1t}}{p_1(2ap_1+b)} + \frac{(1+p_2 c)e^{p2t}}{p_2(2ap_2+b)}\right]. \tag{17.4}$$

2.17. Voltage recovery after short circuit clearance of a synchronous machine without a damper winding

Let's define the equations of the curves of excitation current variation and the voltage buildup on the clamps of the stator winding of the synchronous machine without a damper winding after the clearance of short circuit, if the **rotation velocity** and **voltage [45] interruption** remain unchanged and equal, respectively [illegible] $= 1$.

At the initial moment (at constant three-phase short-circuit), a purely reactive current passes through the stator winding, if the active resistance of the latter is neglected, the maximum value of which is

$$i_d(0) = -I_{m\text{к}}, \tag{18.1}$$

at this

$$i_q(0) = 0. \tag{18.2}$$

The flux linkage of the excitation winding at the initial moment

$$\psi_f(0) = x_f I_f - x_{fa} I_{m\text{к}}, \tag{18.3}$$

where I_f is the initial and constant values of the excitation current.

All the time during the transition process

$$\mathrm{u_f(p) = U_f}. \tag{18.4}$$

After short-circuit disconnection

$$\mathrm{i_d = i_q = i_d(p) = i_q(p) = 0}. \tag{18.5}$$

$$\psi_q = x_q i_q = 0 \tag{18.6}$$

In this case, the equation (62.108) for the excitation current will take the following form

$$i_f(p) = \frac{u_f}{(1+pT_{d0})r_f} + \frac{p\psi_f(0)}{(1+pT_{d0})r_f}. \tag{18.7}$$

The origin of the last equation

$$i_f = I_f\left(1 - e^{-\frac{t}{T_{d0}}}\right) + \frac{\psi_f(0)}{T_{d0}r_f}e^{-\frac{t}{T_{d0}}},$$

or, taking into account (18.3),

$$\mathrm{i_f} = \mathrm{I_f} - \frac{\mathrm{x_{fa}}}{\mathrm{x_f}}\mathrm{I_{м к}}\mathrm{e}^{-\frac{\mathrm{t}}{\mathrm{T_{d0}}}}, \tag{18.8}$$

Initial values of the stator flux linkage:

$$\psi_q(0) = x_q i_q(0) = 0, \tag{18.9}$$

$$\psi_d(0) = x_{af}I_f - x_d I_{м к}. \tag{18.10}$$

Under the conditions of this problem, the equation (62.98) for the stator voltage will take the following form:

$$u_q(p) = \frac{x_{af}[U_f + p\psi_f(0)]}{(1 + pT_{d0})r_f}, \tag{18.11}$$

$$u_d(p) = \left\{-\psi_d(0) + \frac{x_{af}[U_f + p\psi_f(0)]}{(1 + pT_{d0})r_f}\right\}p = p\varphi(p), \tag{18.12}$$

where

$$u_d = \frac{d}{dt}f(t) = p\varphi(p),$$

$$f(t) = \varphi(p). \tag{18.13}$$

Origins of the last equations:

$$u_q = E_m\left(1 - e^{-\frac{t}{T_{d0}}}\right) + \frac{x_{af}\psi_f(0)}{r_f T_{b0}}e^{-\frac{t}{T_{d0}}},$$

$$f(t) = E_m\left(1 - e^{-\frac{t}{T_{d0}}}\right) + \frac{x_{af}\psi_f(0)}{r_f T_{b0}}e^{-\frac{t}{T_{d0}}} - \psi_d(0),$$

$$u_d = \frac{d}{dt}f(t) = \frac{E_m}{T_{d0}}e^{-\frac{t}{T_{d0}}} - \frac{x_{af}\psi_f(0)}{r_f {T^2}_{b0}}e^{-\frac{t}{T_{d0}}}.$$

From (18.7) and the last-mentioned formulas, the instantaneous voltage value on the clamps of the phase *a* is identical

$$u_a = (E_m - x_d{}' I_{м к})\frac{1}{T_{d0}}e^{-\frac{t}{T_{d0}}}\cos\theta_{ad} - \left[E_m\left(1 - e^{-\frac{t}{T_{d0}}}\right) + x_d{}' I_{м к}e^{-\frac{t}{T_{d0}}}\right]\sin\theta_{ad}. \tag{18.16}$$

The first member of the right-hand side, equal to the transformer E. M. F., can be neglected and assume that u_q is the amplitude $u_m$of full-phase voltage. Then

$$u_a = -\left[E_m\left(1 - e^{-\frac{t}{T_{d0}}}\right) + x_d{}' I_{м к}e^{-\frac{t}{T_{d0}}}\right]\sin\theta_{ad}. \tag{18.17}$$

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The curves of amplitude variation of the phase voltage $u_m \approx u_q$ and the excitation current i_f are shown in Figure 18.1.

In a steady-state mode, that is at $t = \infty$ the amplitude of the phase voltage $U_m = E_m$, and at the initial moment, that is at the first moment after switching off the short circuit

$$U_{t0} = x_d{'}I_{m\text{к}}. \tag{18.18}$$

Thus, after short circuit disconnection, that is after the current of the stator, and hence, the m. f. of the armature reaction suddenly decrease to zero, the voltage u_m suddenly increases to $x_d{'}I_{m\text{к}}$ (18.1).

At the moment of current interruption in the stator, in the excitation winding there is a free current inception, which has a direction, as is evident from (18.8), opposite to the steady current of excitation I_f.

As a result of the formulas (18.8) and (18.15), the free current in the excitation winding and the voltage on the stator clamps varies according to the exponential law with a single time constant $T_{d0}$of the excitation winding with an open winding of the stator.

2.18. Voltage recovery after short circuit clearance of a synchronous machine with a damper winding

Let's define the equation of the voltage growth curve on the stator winding clamps after the short circuit clearance if the excitation voltage remains unchanged, that is, if $U_f = U_f(p)$.

As already mentioned above, if neglecting the active resistance of the stator winding, the constant short circuit current $\mathrm{I}_{m\text{к}}$will purely be reactive, i. e.

$$i_{d\infty} = -I_{m\text{к}}; \quad i_q = 0, \tag{19.1}$$

in addition, at short circuit according to (16.3) the longitudinal and lateral flux linkage is equal to zero, i.e.

$$\psi_d(0) = \psi_q(0) = 0. \tag{19.2}$$

Interruption of the short-circuit current can be replaced by the current passing through the stator winding, which is equal to the latter in magnitude and is opposite in direction, that is, the opening is equivalent to the current gain:

$$\begin{gathered} \Delta i_d = I_{m\text{к}} = \Delta i_d(p) \\ \Delta i_q = \Delta i_q(p) = 0. \end{gathered} \tag{19.3}$$

According to (19.2), an increase in the flux linkage after short circuit clearance is equal to the full values of the latter, i.e.

$$\Delta\psi_d = \psi_d; \quad \Delta\psi_q = \psi_q. \tag{19.4}$$

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In addition, since $u_f = const$, then $\Delta u_f = 0$.

Substituting the corresponding magnitudes from (19.1) - (19.4) into (66. 38) gives:

$$\psi_d(p) = x_d(p) I_{m\text{к}}, \quad \psi_q = 0, \tag{19.5}$$

where $x_d(p)$ is determined by (66.17).

We disregard the attenuation of free current in the excitation winding, that is, we put in (19.5) and in (16.17) $r_f = 0$ and $T_{d0} = T_{fm} = \infty$.

Then

$$\psi_d(p) = \frac{x_d{}' + p x_d{}'' T''_{d0}}{1 + pT''_{d0}} I_{m\text{к}}, \tag{19.6}$$

where "T''_{d0} is determined by (16.21). Hence,

$$\psi_d = \left[x_d{}' - (x_d{}' - x_d{}'') e^{-\frac{t}{T''_{d0}}} \right] I_{m\text{к}} \,. \tag{19.7}$$

In view of the free current attenuation in the excitation winding with an approximation sufficient for practice, one can consider

$$\psi_d = \left[x_d - (x_d - x_d{}') e^{-\frac{t}{T_{d0}}} - (x_d{}' - x_d{}'') e^{-\frac{t}{T''_{d0}}} \right] I_{m\text{к}} \,. \tag{19.8}$$

Substituting (19.8) and (19.5) into (15.15) and neglecting the transformer E. M. F., we obtain

$$u_m = u_q = \left[x_d - (x_d - x_d{}') e^{-\frac{t}{T_{d0}}} - (x_d{}' - x_d{}'') e^{-\frac{t}{T''_{d0}}} \right] I_{m\text{к}} \tag{19.9}$$

2.19. Increasing the voltage of a synchronous machine without a damper winding

Let at constant synchronous rotor spinning speed and no-load operation the constant voltage U_f be suddenly applied on the clamping of the excitation winding. We deduce the equations of the curves of excitation current increase, transformer E. M. F., and E. M. F. of the spinning and voltage on the stator winding clamps under zero initial conditions.

From the present conditions it follows:

$$i_d = i_q = i_0 = i_d(p) = i_q(p) = i_d(p) = 0; \tag{20.1}$$

$$\psi_d(0) = \psi_q(0) = \psi_f(0) = 0; \tag{20.2}$$

$$\psi_d = x_{af} i_f; \tag{20.3}$$

$$u_f = u_f(p) U_f; \quad \text{ϱ}_{ad} = \text{ϱ}_r = 1. \tag{20.4}$$

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Substituting the corresponding values from these equations into (62.108) for the excitation current, we find

$$i_f(p) = \frac{1}{1+pT_{d0}} I_f, \tag{20.5}$$

where the fixed value of the excitation current

$$I_f = \frac{U_f}{r_f}. \tag{20.6}$$

The origin of excitation current

$$i_f = I_f\left(1 - e^{-\frac{t}{T_{d0}}}\right). \tag{20.7}$$

The amplitude of the fixed value of E. M. F. in the stator winding

$$E_m = x_{af} I_f. \tag{20.8}$$

The substitution of the expression for i_f from (20.7) and (20.8) into (20.3) gives for the longitudinal flux linkage

$$\psi_d = E_m\left(1 - e^{-\frac{t}{T_{d0}}}\right). \tag{20.9}$$

Since the lateral rotor axis has no windings and the lateral constituent of the stator current is equal to zero, then there will be no lateral flux and lateral flux linkage, i. e. at all-time transient and steady modes

$$\psi_q = 0. \tag{20.10}$$

According to (16. 2) – (16. 5), (20.9) and (20.10) the projections plot the vectors of transformer E. M. F. and E. M. F. of spinning on the axis d, q equal to:

$$\begin{gathered} e_{\text{T}d} = -\frac{d_{\psi d}}{dt} = -\frac{E_m}{T_{d0}} e^{-\frac{t}{T_{d0}}} \\ e_{\text{T}d} = -\frac{d_{\psi d}}{dt} = 0\,, \end{gathered} \tag{20.11}$$

$$\begin{gathered} e_{\text{B}d} = \omega_{ad}\Psi_q = 0, \\ e_{\text{B}q} = -\omega_{ad}\psi_d = -E_m\left(1 - e^{-\frac{t}{T_{d0}}}\right) \end{gathered} \tag{20.12}$$

Since $e_{\text{T}q}$ and $e_{\text{B}d}$ are equal to zero, the plotted vector of the transformer E. M. F. will be directed on the axis d, and the plotted e. m. f. vector of spinning on the axis q, i. e.

$$\overrightarrow{e_{\text{T}d}} = e_{\text{T}d} = e_{\text{T}m},$$
$$\overrightarrow{e_{\text{B}d}} = je_{\text{B}q} = je_{\text{B}m}, \tag{20.13}$$

where $e_{\text{T}m}$, $e_{\text{B}m}$ are the amplitudes of E. M. F.

The amplitude of full phase E. M. F.

$$e_m = \sqrt{e_{\text{T}d}^2 + e_{\text{B}q}^2}\,. \tag{20.14}$$

According to the last-mentioned formulas, the vector diagram of the synchronous generator with the growth of the stator voltage has the form shown in 20.1.

The instantaneous e. m. f. values in phase *a* can be found from equation (18.7):

$$e_a = e_d \cos\theta_{ad} - e_q \sin\theta_{ad} + e_0. \tag{20.15}$$

At $\theta_{ad} = t$ and since $e_0 = 0$, the instantaneous values of both transformer and spinning e. m. f. in phase *a* are equal to:

$$e_{a\text{T}} = -\frac{E_m}{T_{d0}} e^{-\frac{t}{T_{d0}}} \cos t,$$
$$e_{a\text{B}} = E_m\left(1 - e^{-\frac{t}{T_{d0}}}\right)\sin t. \tag{20.16}$$

The curves of amplitudes variation and instantaneous values of transformer and spinning E. M. F. are shown in 20. 2 and 20.3.

Since the stator current is equal to zero, then there is no voltage loss in the stator winding and therefore the phase voltage equals to the sum of E. M. F. with an opposite sign, that is

$$u_a = \frac{E_m}{T_{d0}} e^{-\frac{t}{T_{d0}}} \cos t - E_m\left(1 - e^{-\frac{t}{T_{d0}}}\right)\sin t. \tag{20.17}$$

The magnitude of the time constant of excitation winding with an open stator winding T_{d0}, expressed in radians, has the value of several hundreds, therefore the first constituents of transformer E. M. F. in the stator winding at instant application of the voltage constant on the clamping of excitation winding can be neglected, and only the E. M. F. of spinning can be used, i. e. we can put [47]

$$u_a = -E_m\left(1 - e^{-\frac{t}{T_{d0}}}\right)\sin t. \tag{20.18}$$

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2.20. Enabling two-step excitation voltage

Let us derive an equation for stator voltage and excitation current of the synchronous machine without a damper winding under the following conditions. The stator winding is open, the rotor is spinning with the velocity of $\omega_r = 1$, the rotor rings are fed by excitation voltage U_{f1}, when the time interval t_1 is over, this voltage is instantly increased to U_{f2}.

In this case, one incomplete transient process is replaced by another one. At this, one cannot use the equation of P. Park to investigate another transient process, since the initial conditions are not equal to zero. Also, one cannot use similar equations to increase the magnitudes, since the mode preceding the second transition process was unstable. The problem can be solved by applying the principle of imposing, or by applying the equations of § 62, valid for both transition modes.

From the given conditions it follows:

$$i_d = i_q = i_d(p) = i_q(p) = 0 \ , \psi_q = 0, \tag{21.1}$$

In addition, if $t = 0$ we have i_f *(0) = 0,* <, $\psi_d = 0$, and with $0 < t < t_1$

$$u_f(p) = u_f = U_{f1} = r_f I_{f1} \tag{21.2}$$

when , $t > t_2$

$$u_f = u_f(p) = U_{f2} = r_f I_{f2}, \tag{21.3}$$

where I_{f1}, I_{f2} are excitation currents established at intense u_{f1} and u_{f2}.

The first transient process takes place in the time interval from 0 to t_1 of the radians according to equations (20.7), (20.9) and (20.17). The final values of magnitudes of the first transitional process, the levels of initial values of the corresponding magnitudes of the second process, are determined from the expressions:

$$i_{f2}(0) = I_{f1}\left(1 - e^{-\frac{t}{T_{d0}}}\right). \tag{21.4}$$

$$\psi_d(0) = E_{m1}\left(1 - e^{-\frac{t}{T_{d0}}}\right), \tag{21.5}$$

$$\psi_{f2}(0) = x_f i_{f2}(0) = x_f I_{f1}\left(1 - e^{-\frac{t}{T_{d0}}}\right). \tag{21.6}$$

For the second transition process, the equation (62.98) will have the following form:

$$u_{q2}(p) = \frac{x_{af}[U_{f2} + p\psi_{f2}(0)]}{(1 + pT_{d0})r_f}, \tag{21.7}$$

$$u_{d2}(p) = \left\{-\psi_{d2}(0) + \frac{x_{af}[U_{f2} + p\psi_{f2}(0)]}{(1 + pT_{d0})r_f}\right\} p\,. \tag{21.8}$$

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The amplitude of the fixed value of E. M. F. of the stator in the second transient process

$$\mathrm{E_{m2}} = \mathrm{x_{af}}\frac{\mathrm{U_{f2}}}{\mathrm{r_f}} = \mathrm{x_{af}I_{f2}}. \qquad (21.9)$$

After expanding the operator expression (21.7), we obtain

$$u_{q2} = E_{m2}\left(1 - e^{-\frac{t-t_1}{T_{d0}}}\right) + \frac{x_{af}\psi_{f2}(0)}{r_f T_{d0}} e^{-\frac{t-t_1}{T_{d0}}}.$$

Substituting here the expression for ψ_{f2} (0) from (21.6), we have

$$\mathrm{u_{q2}} = \mathrm{E_{m2}} - (\mathrm{E_{m2}} - \mathrm{E_{m1}})\mathrm{e}^{-\frac{\mathrm{t-t_1}}{\mathrm{T_{d0}}}} - \mathrm{E_{m1}}\mathrm{e}^{-\frac{\mathrm{t}}{\mathrm{T_{d0}}}}. \qquad (21.10)$$

To expand the operational equation (21.8), we represent it in the following form

$$\mathrm{u_{d2}(p)} = \mathrm{p\varphi_2(p)} = \frac{\mathrm{d}}{\mathrm{dt}}\mathrm{f_2(t)}, \qquad (21.11)$$

where

$$\varphi(p) = \frac{E_{m2}}{1+pT_{d0}} + \frac{px_{af}\psi_{f2}(0)}{(1+pT_{d0})r_f} - \psi_{d2}(0).$$

The origin of the last expression

$$f_2(t) = E_{m2} - \left(1 - e^{-\frac{t-t_1}{T_{d0}}}\right) + \frac{x_{af}\psi_{f2}(0)}{r_f T_{d0}} e^{-\frac{t-t_1}{T_{d0}}} - \psi_{d2}(0)$$

or

$$f_2(t) = E_{m2} - (\mathrm{E_{m2}} - \mathrm{E_{m1}})\mathrm{e}^{-\frac{\mathrm{t-t_1}}{\mathrm{T_{d0}}}} - \mathrm{E_{m1}}\mathrm{e}^{-\frac{\mathrm{t}}{\mathrm{T_{d0}}}} - \psi_{d2}(0).$$

Then according to (21.11)

$$u_{d2} = \frac{\mathrm{d}}{\mathrm{dt}}\mathrm{f_2(t)} = \frac{\mathrm{E_{m2}-E_{m1}}}{T_{d0}}\mathrm{e}^{-\frac{\mathrm{t-t_1}}{\mathrm{T_{d0}}}} + \frac{E_{m1}}{T_{d0}}\mathrm{e}^{-\frac{\mathrm{t}}{\mathrm{T_{d0}}}}. \qquad (21.12)$$

Using (20.15), (21.15) and (21.10) for the instantaneous voltage value on the clamps of the phase, we obtain

$$u_a = \frac{1}{T_{d0}}\left[(E_{m2} - E_{m1})e^{-\frac{t-t_1}{T_{d0}}} + E_{m1}e^{-\frac{t}{T_{d0}}}\right]\cos\theta_{ad} -$$

$$-\left[E_{m2} - (E_{m2} - E_{m1})e^{-\frac{t-t_1}{T_{d0}}} - E_{m1}e^{-\frac{t}{T_{d0}}}\right]\sin\theta_{ad}. \qquad (21.13)$$

In these formulas, time *t* and the time constant T_{d0} are expressed in radians. For machines of normal execution of value T_{d0} in radians is equal to a few hundred, so the first member of the right-hand part in (21.13) - the transformer E. M. F. - can be neglected and consider

$$u_a = -\left[E_{m2} - (E_{m2} - E_{m1})e^{-\frac{t-t_1}{T_{d0}}} - E_{m1}e^{-\frac{t}{T_{d0}}}\right] \sin\theta_{ad}. \quad (21.14)$$

The equation of the curve for excitation current variation is found in (62.108), which in conditions of this problem will have the following form:

$$i_{f2}(p) = \frac{1}{(1+pT_{d0})r_f}\left[U_{f2} + p\psi_{f0}(0)\right]$$

the origin of the excitation current

$$i_{f2} = I_{f2}\left(1 - e^{-\frac{t-t_1}{T_{d0}}}\right) + \frac{\psi_{f2}(0)}{r_f + T_{d0}} e^{-\frac{t-t_1}{T_{d0}}}$$

or

$$i_{f2} = I_{f2} - \left(I_{f2} - I_{f1}\right)\mathrm{e}^{-\frac{t-t_1}{T_{d0}}} - I_{f1}\mathrm{e}^{-\frac{t}{T_{d0}}}. \quad (21.15)$$

As follows from the comparison (21.14) with (21.15), the excitation current and the amplitude of the phase voltage vary by one law. The curves for changing the amplitude of the phase voltage $u_m \approx u_q$of the excitation current are shown in Figure 21.1.

2.21. Increase in the voltage of a synchronous machine with a damper winding

Let a synchronous machine with a damper winding and an open winding of the stator spins at a synchronous speed. On the clamp of the winding, a constant voltage is suddenly applied. We deduce the equation of the curves of the excitation current and voltage increase on the clamps of the stator winding under zero initial conditions [48].

From the given conditions it follows:

$$i_d = i_q = i_0 = i_d(p) = i_q(p) = i_0(p) = 0;$$
$$u_f(p) = U_f; \quad \psi_d(0) = 0; \quad \psi_q = \psi_q(p) = \psi_q(0) = 0. \quad (22.1)$$

At zero initial conditions and $i_d = 0$ equations for the longitudinal flux linkage will have the following form:

$$\psi_d(p) = \frac{1 + p(T_{1d0} - T_{1Md})}{T_{d0}T''_{d0}p^2 + (T_{1d} + T_{d0})p + 1}\frac{x_{af}U_f}{u_f}. \quad (22.2)$$

Using the decomposition theorem, we expand this operator equation.

Let's denote

$$\mathrm{M}\ (\mathrm{p}) = 1 + \mathrm{cp},$$
$$\mathrm{N}\ (\mathrm{p}) = \mathrm{ap}^2 + \mathrm{bp} + 1 \quad (22.3)$$

where

$$\mathrm{c} = T_{1d0} - T_{1\mathrm{M}d},$$
$$a = T_{d0}T''_{d0}, \quad (22.4)$$
$$b = T_{d0} + T_{1d}$$
$$N'\ (p) = 2ap + b.$$

(22.5)

The roots of the equation $N(p) = 0$ are determined from the expression

$$p_{1,2} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{1}{a}}\ . \quad (22.6)$$

Since

$$x_{af}\frac{U_f}{r_f} = x_{af}I_f = E_m,$$

then according to formula (26) in appendix 6 for the origin of the longitudinal flux linkage, we can write the equation:

$$\psi_d = E_m\left[1 + \frac{(1+p_1c)e^{p1t}}{p_1(2ap_1+b)} + \frac{(1+p_2c)e^{p2t}}{p_2(2ap_2+b)}\right]. \quad (22.7)$$

If the roots are real, then, as it can be seen from (22.6), both of them are negative, at that p_1 is much higher than p_2. Thus, as follows from (22.7), in the period of E. M. F. rise the longitudinal flux linkage consists of three components:

a) the constant E_mb) rapidly fading, directly proportional to e^{p1t}

c) slowly decays, directly proportional to e^{p2t}.

If one neglects the diffusion of excitation and damper windings and, that is to put it

$$x_f = x_{f\mathrm{M}},\ \ x_{1d} = x_{1\mathrm{M}d}\ ,$$
$$T_{d0} = T_{f\mathrm{M}},\ T_{1d} = T_{1\mathrm{M}d} \quad (22.8)$$

$$T_{d0}T''_{d0} = T_{d0}T_{1d} - T_{f\mathrm{M}}T_{1\mathrm{M}d} = 0.$$

then equation (22.2) in the operator and the original form will take the following form:

$$\psi_d(p) = E_m\frac{1}{1 + (T_{d0} + T_{1d})p},$$
$$\psi_d(p) = E_m\left(1 - e^{-\frac{t}{T_{d0}+T_{1d}}}\right). \quad (22.9)$$

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Since $\psi_q = 0$, then the amplitudes of transformer and spinning E. M. F. in accordance with (16.2) and (16.5) will be equal to:

$$e_{\text{т}} = e_{\text{т}d} = -\frac{E_m}{T_{d0} + T_{1d}} e^{-\frac{t}{T_{d0}+T_{1d}}},$$

$$e_{\text{в}} = e_{\text{в}q} = -\psi_d = -E_m\left(1 - e^{-\frac{t}{T_{d0}+T_{1d}}}\right). \tag{22.10}$$

Since $e_{\text{т}q}$ and $e_{\text{в}d}$ are equal to zero, the projections depicting the vector of a complete E. M. F. of the stator winding on the axis *d* and *q* will be:

$$e_{d=}e_{\text{т}d} = e_{\text{т}} \ ,$$

$$e_q = e_{\text{в}q} = e_{\text{в}}. \tag{21.11}$$

At no-load operation $u_d = -e_d$, $u_q = -e_q$ therefore at $\theta_{ad} = t$, when using (20.15), (22.10) and (22.11) for the instantaneous value of the voltage of phase *a*, we obtain

$$u_a = \frac{E_m}{T_{d0}+T_{1d}} e^{-\frac{t}{T_{d0}+T_{1d}}} cos\, t - E_m\left(1 - e^{-\frac{t}{T_{d0}+T_{1d}}}\right) sin\, t. \tag{22.12}$$

Like in a machine without a damper winding, the transformer E. M. F. compared with E. M. F. of spinning is minimal, and therefore the first components in (22.12), without making a severe error, can be neglected, that is, to assume that

$$u_a = -E_m\left(1 - e^{-\frac{t}{T_{d0}+T_{1d}}}\right)\sin t \tag{22.13}$$

Consequently, in the absence of diffusion in both the excitation and damper windings the growth of the phase voltage occurs as well as the growth of E. M. F. of spinning under the exponential law with the time constant, which is equal to the sum of the time constant of excitation and damper windings with the open winding of the stator. At zero initial conditions and $i_d = 0$ of equation (66. 9) and (66. 10) for excitation and damper currents will take the following form:

$$i_f(p) = \frac{(r_{1d} + px_{1d})U_f}{A\,(p)},$$

$$i_{1d}(p) = -\frac{px_{1f}U_f}{A\,(p)},$$

where *A(p)* is determined by the formula (66, 11).

If neglecting the excitation and damper windings, that is, if we take the equalities (22.8), then

$$\mathrm{A(p) = [(T_{d0} + T_{1d})p + 1]r_1 r_{1d}}$$

and

$$i_f(p) = \frac{1 + pT_{1d}}{1 + (T_{d0} + T_{1d})p} I_f,$$

$$i_{id}(p) = \frac{px_{1f}I_f}{[1 + (T_{d0} + T_{1d})p]r_{1d}}. \tag{22.14}$$

Expanding the last two operator expressions, we deduce:

$$i_f = \left(1 - \frac{T_{d0}}{T_{d0}+T_{1d}} e^{-\frac{t}{T_{d0}+T_{1d}}}\right) I_f\,, \tag{22.15}$$

$$i_{id} = \frac{x_{1f} I_f}{(T_{d0}+T_{1d})r_{1d}} e^{-\frac{t}{T_{d0}+T_{1d}}}\,. \tag{22.16}$$

As follows from the curves of both currents i_f and i_{id} shown in 22.1, in the absence of diffusion of the excitation and damper windings, acting on each other like bifilars, instantaneous increases in the currents in the latter at the initial moment are caused by the magnitudes:

$$2.\Delta I_f = \frac{T_{1d}}{T_{d0}+T_{1d}} I_f,$$

$$\Delta I_{1d} = \frac{x_{1f}}{(T_{d0} + T_{1d})r_{1d}} I_f \tag{22.17}$$

2.22. General operating expressions for currents of a sudden three-phase short circuit of a synchronous machine at $n = const$ and $U_f = const$

An analytical study of a sudden three-phase short circuit of a synchronous machine in the general form taking into account the active resistance of windings, even at constant speed of the rotor, presents significant mathematical difficulties.

To simplify analytical research, there are usually a number of assumptions made. In a number of cases, these assumptions are not only reflected in the accuracy of calculations, but also distort the actual picture of transition process at short circuit of the synchronous machine. For example, the formulas for short circuit currents do not reflect the slow spinning of the magnetic field created by the so-called aperiodic components in the stator, and do not have approximately twice the frequency.

In an effort to avoid these disadvantages, a combination of assumptions has been applied that make it possible without unnecessary complications of calculations to take into account both the active resistances of the machine and the load and obtain formulas that reflect the physical process of the short circuit of synchronous machine as well as suitable for practical application in calculations [49].

Let us consider the case of a three-phase short circuit stator winding of synchronous machine, which operated until the moment of short circuit in a steady-state mode of symmetric loading. We assume the rotor spinning speed as synchronous, and the excitation voltage as constant in all terms of short circuit. Under accepted conditions, equations (66.43) will be valid for increasing the stator currents.

In case of a sudden three-phase short circuit, the phase voltage of the network and, hence, the longitudinal and lateral components i. b. of the voltage u_d and u_q suddenly drop to zero.

If before the short circuit the constituents i. b. of the stator voltage were U_{md0} and U_{mq0} , then the effect of short circuit, as is known, can be obtained by sudden application of phase voltages on the stator clamp that form I. B. which are equal to $-\mathrm{U}_{\mathrm{md0}}$ and $-U_{mq0}$.

At the same time, currents in phases at short circuit can be found by adding the currents $\mathrm{I}_{\mathrm{md0}}$ and $\mathrm{I}_{\mathrm{mq0}}$, those that existed before the short circuit, with currents $\Delta\ i_d$ and Δi_q, caused by the sudden application of voltages $-U_{md0}$ and $-U_{mq0}$.

Thus, to describe the short circuit currents, the following formulas can be written:

$$\begin{aligned} \mathrm{i_d(p)} &= \Delta \mathrm{i_d(p)} + \mathrm{I_{md0}}, \\ \mathrm{i_q(p)} &= \Delta \mathrm{i_q(p)} + \mathrm{I_{mq0}}. \end{aligned} \tag{23.1}$$

As in the case under consideration:

$$\begin{aligned} \Delta \mathrm{u_d(p)} &= -\mathrm{U_{mq0}}, \\ \Delta \mathrm{u_q(p)} &= -\mathrm{U_{mq0}}. \end{aligned} \tag{23.2}$$

then from (23.1) and (66.43) we obtain:

$$i_d(p) = -\frac{Z'_q U_{md0} + X_q(p) U_{mq0}}{Z_d' Z_q' + X_d(p) X_q(p)} + I_{md0}\ , \tag{23.3}$$

$$i_q(p) = -\frac{Z'_d U_{mq0} - X_d(p) U_{md0}}{Z_d' Z_q' + X_d(p) X_q(p)} + I_{md0}\ ,$$

The amplitudes of currents of the steady-state mode preceding the short circuit can be determined on the basis of (4.2) by the following formulas:

$$\begin{aligned} I_{md0} &= \frac{rU_{md0} + x_q U_{mq0} - x_q E_m}{r^2 + x_d x_q}, \\ I_{mq0} &= \frac{rU_{mq0} + x_d U_{md0} - rE_m}{r^2 + x_d x_q}, \end{aligned} \tag{23.4}$$

At no-load operation, the flux of the rotor Φ_f induces in the stator phases the e. m. f., whose vector $\bar{E}$ lagging behind from Φ_f by 90°, is directed in the negative direction of the q axis (23.1). The voltage vector on the clamps of the stator winding $\overline{U_0}$ at no-load operation, being equal to and opposite to the vector of e. m. f. $\overline{E}$, is directed to the positive side of the q axis

So, at no-load operation

$$\begin{gathered} U_{md0} = 0 \\ U_{mq0} = E_m , \\ I_{md0} = I_{mq0} = 0 \end{gathered} \tag{23.5}$$

where E_m is the positive scalar value equal to the amplitude of E. M. F. in the stator phase.

Under short circuit at no-load operation, the formulas for the stator currents will be obtained by substituting into (23.3) the values of voltages, currents and resistances from (23.5) and (16.42); then we obtain:

$$i_d(p) = -\frac{X_q(p)E_m}{X_d(p)\cdot X_q(p)p^2+[X_d(p)+X_q(p)]rp+r^2+X_d(p)X_q(p)}, \tag{23.6}$$

$$i_q(p) = \frac{[pX_d(p)+r]E_m}{X_d(p)\cdot X_q(p)p^2+[X_d(p)+X_q(p)]rp+r^2+X_d(p)X_q(p)}.$$

Finding the origins of the longitudinal and lateral components i. b. of the current i_d and i_q, it is possible to determine the instantaneous values of the short circuit current in the phase *a* by formula (18.7), putting into it $\theta_{ax} = t + \theta_0$, $i_0 = 0$ and replacing the index *x* by *d*, that is,

$$i_a = i_d\cos(t+\theta_0) - i_q\sin(t+\theta_0)\,. \tag{23.7}$$

2.23. Equations for short-circuit currents of a synchronous machine without a damper winding

Joint solution at $n = const\ u,\ U_f = const$

If the rotor has only an excitation winding, located on the longitudinal axis, according to (62. 18)

$$X_d(p) = \frac{x_d + p x_d{}' T_{d0}}{1+pT_{d0}}, \tag{24.1}$$

$$X_q(p) = x_q. \tag{24.2}$$

The denominator of operator expressions (23.3) and (23.6) in this case equals

$$D(p) = \frac{x_q x_d' T_{d0}}{1+pT_{d0}} N(p), \tag{24.3}$$

where

$$N(p) = p^3 + \left(\frac{r}{x_q} + \frac{x_d + rT_{d0}}{x_d' T_{d0}}\right) p^2 + \left[\frac{r(x_d + x_q + rT_{d0})}{x_q x_d' T_{d0}} + 1\right] p + \frac{r^2 + x_q x_d}{x_q x_d' T_{d0}}. \tag{24.4}$$

Taking into account (24.3) and (16.42), we write (23.3) in the following form:

$$i_d(p) = -\frac{(1+pT_{d0})[px_q + r)U_{md0} + x_q U_{mq0}]}{x_q x_q' T_{d0} N(p)} + I_{md0}\,,$$

$$i_q(p) = -\frac{[x_d' T_{d0} p^2 + (x_d + rT_{d0})p + r]U_{mq0} - (x_d' p T_{d0} + x_d) U_{md0}}{x_q x_q' N(p)} + I_{mq0}. \tag{24.5}$$

In the last two equations, the second items of the right-hand parts are the currents of the steady-state mode before the short circuit.

Steady currents at short circuit can be found by putting into the indicated equations $p = 0$. In this case, taking into account (23.4) and (24.4), we obtain:

$$I_{\text{к}md} = -\frac{x_q E_m}{r^2 + x_d x_q}\,, \tag{24.6}$$

$$I_{\text{к}md} = -\frac{rE_m}{r^2 + x_d x_q}\,.$$

If we put the active resistance of the stator winding equal to zero, we obtain a well-known formula for the steady short-circuit current

$$I_{\text{к}m} = -I_{\text{к}md} = \frac{E_m}{x_d}\,. \tag{24.7}$$

The instantaneous value of steady-state short circuit current in phase a, taking into account the active resistance of the stator winding, has been found, substituting the expressions for currents from (24.6) into (23.7):

$$i_{\text{к}\infty} = -\frac{E_m}{r^2 + x_d x_q}\left[x_q \cos(t + \theta_0) - r \sin(t + \theta_0)\right]. \tag{24.8}$$

To expand the operating items of the right-hand parts of equations (24.5), we can use the expansion theorem. Finding three roots p_1, p_2 and p_3 of the cubic equation (24.4)

$$\mathrm{N}\,(\mathrm{p}) = 0 \tag{24.9}$$

So, taking into account formula (24.6), we obtain the following expressions for currents:

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$$i_d = -\frac{x_q E_m}{r^2 + x_q x_d} - \sum_{n=1}^{n=3} \frac{Ж_{кd}}{x_q x_d{}' p_n T_{d0} N'(p_n)} e^{-p_n t},$$

$$i_q = -\frac{r E_m}{r^2 + x_q x_d} - \sum_{n=1}^{n=3} \frac{Ж_{кd}}{p_n x_q x_d{}' T_{d0} N'(p_n)} e^{-p_n t} \quad (24.10)$$

where

$$Ж_{кd} = (1 + p_n T_{d0})\left[(r + p_n x_q) U_{md0} + x_q U_{mq0}\right],$$

$$Ж_{кq} = [p_n x_d{}' T_{d0} + (x_d + r T_{d0}) p_n + r] U_{mq0} - (p_n x_d{}' T_{d0} + x_d) U_{md0} .$$

The difficulties of practical use of the obtained equations are to determine the roots of the cubic equation (24.9) and the complexity of algebraic transformations, especially when solving a problem in general form. The numerical solution of equation (24.9) is simpler and can be performed in the following sequence [49]. Let's denote :

$$a = \frac{r}{x_q} + \frac{x_d + r T_{d0}}{x_d{}' T_{d0}},$$

$$b = 1 + \frac{r(x_q + x_d + r T_{d0})}{x_q x_d{}' T_{d0}}, \quad (24.11)$$

$$c = \frac{r^2 + x_q x_d}{x_q x_d{}' T_{d0}}.$$

Then the equation (24.9) will take the form

$$p^3 + a p^2 + b p + c = 0 \quad (24.12)$$

To solve this cubic equation, we introduce the notation:

$$m = -\frac{a^2}{3} + b , \quad (24.13)$$

$$q = \frac{2a^3}{27} - \frac{ab}{3} + c.$$

Then, we'll consider the expression

$$\frac{q^2}{4} + \frac{m^3}{27} = \frac{1}{4}\left(\frac{2a^3}{27} - \frac{ab}{3} + c\right)^2 + \frac{1}{27}\left(-\frac{a^2}{3} + b\right)^3. \quad (24.14)$$

Obtaining positive numerical values for both parts of this equality (after performing the shown actions) is provided with

$$-\frac{a^2}{3} + b > 0 . \quad (24.15)$$

If the active resistance of the stator winding is zero

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$$(r = 0),\ \text{то}\ \frac{a^2}{3} = \frac{1}{3}\left(\frac{x_d}{x_d' T_{d0}}\right)^2 \text{ i } b = 1.$$

Since in normal-execution machines T_d0 is small in radians, and x_d / (x_d ') usually does not exceed 10, inequality (24.15) is valid, and therefore

$$\frac{q^2}{4} + \frac{m^3}{26} > 0.$$

This inequality takes place in valid machines at r ≠ 0. Let's determine the real values of the radicals:

$$u_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{m^3}{27}}},$$

$$v_1 = \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{m_3}{27}}}. \qquad (24.16)$$

In these formulas, the subjective expression in the square root is positive and, therefore, the square root itself is material. For u_1 and v_1, values that satisfy the condition must be taken

$$u_1 v_1 = -\frac{m}{3}, \qquad (24.17)$$

so by defining u_1, one can define $v_1$by the formula

$$v_1 = -\frac{m}{3u_1}. \qquad (24.18)$$

The roots of the cubic equation (24.12) have the form:

$$p_1 = u_1 + v_1 - \frac{a}{3},$$

$$p_2 = -\frac{1}{2}(u_1 + v_1) - \frac{a}{3} + j\frac{\sqrt{3}}{2}(u_1 - v_1),$$

$$p_3 = -\frac{1}{2}(u_1 + v_1) - \frac{a}{3} - j\frac{\sqrt{3}}{2}(u_1 - v_1). \qquad (24.19)$$

From the obtained expressions it follows that one of the roots is real, and the others two are imaginary related. As it is proved in the theory of the operative calculus, in this case the originals of the functions i_d and i_q will each have three components:

a) stable non-exhausting, b) aperiodic, decaying with constant time

$$T_d{}' = -\frac{1}{p_1} = -\frac{1}{u_1+v_1-\frac{a}{3}}, \qquad (24.20)$$

c) periodic, decaying with a constant time equal to the value of the inverse actual part of the roots p_2 and p_3, that is,

$$T_a{}' = \frac{1}{\frac{1}{2}(u_1+v_1)+\frac{1}{3}a}. \qquad (24.21)$$

Thus, the components i. в. of the short circuit current can be represented in the following general form:

$$i_d = A_{1d} + A_{2d}e^{-\frac{t}{T_d{}'}} + A_{3d}e^{-\frac{t}{T_a{}'}}\cos(\omega_к t + \varphi_d)\,,$$
$$i_q = A_{1q} + A_{2q}e^{-\frac{t}{T_d{}'}} + A_{3q}e^{-\frac{t}{T_a{}'}}\cos(\omega_к t + \varphi_q)\,, \qquad (24.22)$$

where $A_{1d}, A_{2d}, A_{3d}, A_{1q}, A_{2q}, A_{3q}, \omega_к, \varphi_d$, i φ_q are constants dependent on machine parameters and initial conditions. At the same time, in the machines of general industrial execution $\omega_к \approx 1$.

Substituting the expressions for i_d and i_q from (24.22) in (23.7), we obtain for the instantaneous value of the short circuit current in the phase the expression

$$i_a = \left[A_{1d}\cos(t+\theta_0) - A_{1q}\sin(t+\theta_0)\right] +$$
$$+e^{-\frac{t}{T_d{}'}}\left[A_{2d}\cos(t+\theta_0) - A_{2q}\sin(t+\theta_0)\right] +$$
$$+\frac{1}{2}e^{-\frac{t}{T_a{}'}}\left[A_{3d}\cos(t-\omega_к t+\theta_0-\varphi_d) - A_{3q}\sin(t—\omega_к t+\theta_0-\varphi_d)\right] +$$
$$+\frac{1}{2}e^{-\frac{t}{T_a{}'}}\left[A_{3d}\cos(t+\omega_к t+\theta_0++\varphi_d) - A_{3q}\sin(t+\omega_к t+\theta_0+\varphi_d)\right]. \qquad (24.23)$$

Consequently, the phase short circuit curren. has four components:

a) constant short circuit current of nominal frequency determined from (24.7) and equal to

$$i_{к\infty} = A_{1d}\cos\,(t+\theta_{0-A_{1q}\sin}(t+\theta_0)\;; \qquad (24.24)$$

b) transient short circuit current with rated frequency, fading with constant time $T_d{}'$

$$\Delta i_к{}' = \left[A_{2d}\,cos(t+\theta_0) - A_{2q}\,sin(t+\theta_0\right]e^{-\frac{t}{T_d{}'}}; \qquad (24.25)$$

c) practically the aperiodic short circuit current of the crystalline state with a frequency close to zero and fading with a constant time T_a '

$$i_{\kappa a} = \{A_{3d} cos[(1-\omega_{\kappa})e + \theta_0 - \varphi_d] - A_{3q} sin[(1-\omega_{\kappa})t + \theta_0 - \varphi_q]\}\frac{1}{2}e^{-\frac{t}{Ta'}} \quad (24.26)$$

d) periodic short circuit current of practically double frequency (1 + ω_k) ω_c, fading with constant time T_a '

$$i_{\kappa 2} = \{A_{3d} cos[(1+\omega_{\kappa})t + \theta_0 + \varphi_d] - A_{3q} sin[(1++\omega_{\kappa})t + \theta_0 + \varphi_q]\}\frac{1}{2}e^{-\frac{t}{Ta'}} \quad (24.27)$$

Equation with $r \neq 0,\ r_f = 0,\ n = const\ i\ U_f = const$

Let's expand the operator expressions (24.5) for the stator currents at the short circuit three-phase of a synchronous machine without a damper winding at no-load operation, accepting during the short circuit the rotor speed to be synchronous, the excitation voltage constant, the constant time of the excitation winding T_d0 is equal to infinity and, consequently, r_f = 0. The last assumption will allow us to simply determine precisely the constant time of the winding of the stator T_a 'with a short-circuit excitation winding and an equation for the aperiodic component of the short circuit current. At given conditions at the time that preceded the short circuit

$$I_{mq0} = I_{md0} = 0$$

and with T_d0 = ∞ on the basis of (24.1) and (24.2)

$$X_q(p) = x_q, \qquad X_d(p) = x_d'. \quad (24.28)$$

Formulas (23.6) will take the form:

$$i_d(p) = -\frac{I'_{md}}{p^2 + r\frac{x_q + x_d'}{x_q x_d'}p + \left(1 + \frac{r^2}{x_q x_d'}\right)},$$

$$i_q(p) = -\frac{I'_{md}}{x_q} - \frac{r + p x_d'}{p^2 + r\frac{x_q + x_d'}{x_q x_d'}p + \left(1 + \frac{r^2}{x_q x_d'}\right)}, \quad (24.29)$$

where

$$I'_{md} = \frac{E_m}{x_d'} \quad (24.30)$$

is the amplitude of the transient short circuit current. The roots of operator denominators in (24.29) are equal to

$$p_{1,2} = -\alpha_a' \pm j\omega_{\kappa}\ , \quad (24.31)$$

where

$$\omega_{\kappa} = \sqrt{1 - \left(\frac{x_d' - x_q}{2x_q x_d'}\right)^2 r^2}\ , \quad (24.32)$$

$$\alpha_a{}' = \frac{x_q + x_d{}'}{2x_q x_d{}'} r = \frac{1}{T_a{}'} \,. \tag{24.33}$$

In most cases it is quite possible to take ω_k = 1 with the accuracy that is quite sufficient for practice. At this

$$\begin{gathered} p_1 = -\alpha_a{}' + j, \\ p_2 = -\alpha_a{}' - j. \end{gathered} \tag{24.34}$$

Using the formula of decomposition [see (28) in appendix 6] we obtain:

$$\begin{gathered} i_d = i_{d\infty} + 2\sqrt{A_d^2 + B_d^2} e^{-\alpha_{a'} t} \cos(\omega_\kappa t + \varphi_d), \\ i_q = i_{q\infty} + 2\sqrt{A_q^2 + B_q^2} e^{-\alpha_{a'} t} \cos(\omega_\kappa t + \varphi_q), \end{gathered} \tag{24.35}$$

Where $i_{d\infty}$, $i_{q\infty}$ are fixed short circuit currents

$$\begin{gathered} i_{d\infty} = -\frac{x_q x_d{}'}{r^2 + x_q x_d} I'_{md}, \\ i_{q\infty} = -\frac{r x_d{}'}{r^2 + x_q x_d} I'_{md}. \end{gathered} \tag{24.36}$$

Neglecting the influence of the active resistance of the stator winding on the values A_d, B_d, A_q, B_q, φ_d and φ_q, we find for them the values:

$$\begin{gathered} A_d = -\frac{I'_{md}}{2}. \\ B_d = 0 \,, \end{gathered} \tag{24.37}$$

$$\begin{gathered} A_q = 0 \,, \\ B_q = -\frac{E_m}{2x_q} \,; \end{gathered} \tag{24.38}$$

$$\begin{gathered} \varphi_d = 0, \\ \varphi_q = -90°. \end{gathered} \tag{24.39}$$

Substituting the expressions for currents and their constants (24.36) - (24.39) into (24.35), we obtain:

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$$i_d = -I'_{md}\left(\frac{x_q x_d'}{r^2 + x_q x_d'} - e^{-\frac{t}{T_a'}} \cos \omega_{\text{к}} t\right),$$

$$i_q = -I'_{md}\left(\frac{r x_d'}{r^2 + x_q x_d'} + \frac{x_d'}{x_q} e^{-\frac{t}{T_a'}} \sin \omega_{\text{к}} t\right). \tag{24.40}$$

Substituting the resulting expressions for i_d and i_q in (23.7) and taking into account (24.30), we find for the instantaneous value of the short circuit current in phase *U* the following expression

$$i_a = -E_m \left\{ \left[\frac{x_q}{r^2 + x_q x_d'} \cos(t + \theta_0) - \frac{r}{r^2 + x_q x_d'} \sin(r + +\theta_0) \right] - \right.$$
$$- \left(\frac{1}{2x_q} + \frac{1}{2x_d'} \right) e^{-\frac{t}{T_a'}} \cos[(\omega_{\text{к}} - 1)t + +\theta_0] +$$
$$\left. + \left(\frac{1}{2x_q} - \frac{1}{2x_d'} \right) e^{-\frac{t}{T_a'}} cos[(\omega_{\text{к}} + 1)t + \theta_0] \right\} \tag{24.41}$$

The accuracy of calculation of the members of the obtained equation is notidentical: while the first term in the curly brackets is determined taking into account the active resistance of the stator winding, the coefficients in round brackets before the other two terms were determined at r = 0. At the initial moment, the short circuit current, calculated by formula (24.41), will be equal to zero only if in the whole of this formula we put r = 0.

From the expression (24.41) it follows that the short circuit current in the phase *a* of the synchronous machine without a damper winding, r_f = 0 and T_d0 = ∞ consists of three components. Let us consider each of them.

a) Symmetrical non-damped short circuit current of the steady-state nominal frequency mode

$$i'_{\text{к}\infty} = -\frac{E_m}{r^2 + x_q x_d'} \left[x_q \cos(t + \theta_0) - r \sin(t + \theta_0) \right] \tag{24.42}$$

or with $r = 0$

$$i'_{\text{к}\infty} = -\frac{E_m}{x_d'} cos(t + \theta_0).$$

In the actual machines, the active resistance of the excitation winding is not equal to zero, and therefore this component will fade to the value of the steady-state current, which is determined by (24.8).

Subtracting from (24.42) the equation (24.8), we obtain an expression for the transitive component of the short circuit current in the phase *U*, taking into account the stator's active resistance, but ignoring the attenuation [50]:

$$\Delta i_{\text{к}}' = -\frac{x_q (x_d - x_d') E_m}{(r^2 + x_q x_d')(r^2 + x_q x_d)} \left[x_q \cos(t + \theta_0) - r \sin(t + \theta_0) \right]. \tag{24.43}$$

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At r = 0

$$\Delta i_{к}' = -\left(\frac{1}{x_d'} - \frac{1}{x_d}\right) E_m \cos(t + \theta_0). \qquad (24.44)$$

In the real machine, the transient short circuit current component fails with a time constant T_a', the value of which is determined below.

b) Current determined by the expression

$$i'_{кa} = -\left(\frac{1}{2x_q} + \frac{1}{2x_d'}\right) E_m e^{-\frac{t}{T_a'}} \cos[(\omega_1 - 1)t + \theta_0], \qquad (24.45)$$

fades with a time constant T_d '(24.33) and changes with a very small frequency, which is equal to (24.32)

$$\omega_{кa} = (\omega_к - 1)\omega_c = \left[\sqrt{1 - \left(\frac{x_d' - x_q}{2x_q x_d'}\right)^2 r^2 - 1}\right] \omega_c \approx 0. \qquad (24.46)$$

Thus, the current $i'_{кa}$ is practically aperiodic and its variation with frequency ω_кa is usually not taken into account.

c) symmetrical, attenuating with the time constant T_a' current of practically double frequency

$$i'_{к2} = -\left(\frac{1}{2x_q} - \frac{1}{2x_d'}\right) E_m e^{-\frac{t}{T_a'}} \cos[(\omega_1 + 1)t + \theta_0] \approx$$

$$\approx -\left(\frac{1}{2x_q} - \frac{1}{2x_d'}\right) E_m e^{-\frac{t}{T_a'}} \cos[2t + \theta_0]. \qquad (24.47)$$

The equation at r ≠ 0, r_f = 0, n = const and U_f = const

As it turned out in section 23, it was considered that the effect of short circuit on no-load operation can be obtained by attaching to the stator clamps the voltages

$$-U_{md0} = 0, \quad -U_{mq0} = -E_m. \qquad (24.48)$$

Substituting these values forvoltages, as well as comparing$r = 0$, $I_{md0}0$ and $I_{mq0} = 0$ in (24.5) and (24.4), we obtain in this case for the short circuit current the expression:

$$i_d(p) = -\frac{(1 + pT_{d0})E_m}{(p^3 + \alpha_d' p^2 + p + \alpha_d')x_d' T_{d0}},$$

$$i_q(p) = -\frac{(x_d' p T_{d0} + x_d)pE_m}{(p^3 + \alpha_d' p^2 + p + \alpha_d')x_q x_d' T_{d0}}, \qquad (24.49)$$

where

$$\alpha_d' = \frac{1}{T_d'} = \frac{x_d}{x_d' T_{d0}'}. \qquad (24.50)$$

We find the roots of the cubic equation

$$N(p) = p^3 + \alpha_d' p^2 + p + \alpha_d' = 0. \qquad (24.51)$$

We'll introduce the designation:

$$m = -\frac{\alpha'^2_d}{3} + 1\,,$$

$$q = \frac{2}{27}\alpha_{d^3} + \frac{2}{3}\alpha_d'\,.$$

At the same time according to (24.16) and (24.18)

$$u_1 = \sqrt{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{m^3}{27}}} = -\frac{\alpha_d'}{3} + \frac{1}{\sqrt{3}},$$

$$v_1 = -\frac{m}{3u_1} = \sqrt{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{m^3}{27}}} = -\frac{\alpha_d'}{3} - \frac{1}{\sqrt{3}}$$

According to (24.19), the roots of the cubic equation (24.51) are equal to:

$$p_1 = -\alpha_d',$$

$$p_2 = +j, \qquad (24.52)$$

$$p_3 = -j.$$

Using while expanding the operating expression (24.49) the assumptions

$$1 + \frac{1}{T'^2_d} \approx 1,$$

$$1 + T^2{}_{d0} \approx T^2{}_{d0}, \qquad (24.53)$$

we deduce:

$$i_d = -\left[\frac{1}{x_d} + \left(\frac{1}{x_d'} - \frac{1}{x_d}\right)e^{-\frac{t}{T_d'}} - \frac{1}{x_d'}\cos t\right]E_m\;,$$

$$i_q = -\frac{E_m}{x_q}\sin t\,. \qquad (24.54)$$

The instantaneous value of the current in phase U will be found by substituting expression (23.7) i_d and i_q from (24.54):

$$i_a = -E_m\left[\frac{1}{x_d}\cos(t+\theta_0) + \left(\frac{1}{x_d{'}} - \frac{1}{x_d}\right)e^{-\frac{t}{T_d{'}}}\cos(t+\theta_0) - \right.$$

$$\left. -\left(\frac{1}{2x_d{'}} + \frac{1}{2x_q}\right)\cos\theta_0 + \left(\frac{1}{2x_q} - \frac{1}{2x_d{'}}\right)\cos(2t+\theta_0)\right]. \qquad (24.55)$$

As it can be seen from the last expression that with the admitted assumption $r_s = 0$ the short circuit current consists of four components:

a) the fixed periodic short circuit current.; b) fading with the time constant $T_d{'}$ of the transient periodic current; c) the constant current component; d) non-disipative current, which varies with double frequency.

The last two components are damped, since $r_s = 0$ (according to our assumption, the active resistance of the stator winding is zero).

A complete expression for the current of a sudden three-phase short circuit

From the consideration of the preceding cases we can conclude that the general expression for the stator current in case of sudden three-phase short circuit of the synchronous machine without a damper winding includes the following four components:

a) fixed symmetrical non-damped short circuit current of synchronous frequency $i_{к\infty}$, which is determined by (24.8);

b) transient symmetric component $\Delta i_к{'}$ of synchronous frequency, decaying with the constant time $T_d{'}$, determined by (24.43);

c) practically aperiodic current $i'_{кa}$, fading with the time constant $T_a{'}$, determined by (24.45);

d) the symmetric current $i'_{кa}$ is practically of double frequency, fading with the time constant $T_a{'}$, which is determined by (24.47).

Formulating equations (24.8), (24.43), (24.47) and (24.45), for the current of phase a, we obtain the following expression [51]:

$$i_к = i_{к\infty} + \Delta i_к{'} + i'_{к2} + i'_{кa} = -\frac{E_m}{r^2 + x_q x_d}\times$$

$$\times\left[x_q\cos(t+\theta_0) - r\sin(t+\theta_0)\right] - \frac{x_q(x_d - x_d{'})E_m e^{-\frac{t}{T_d{'}}}}{(r^2 + x_q x_d{'})(r^2 + x_q x_d)}\times$$

$$\times\left[x_q\cos(t+\theta_0) - r\sin(t+\theta_0)\right] - \left(\frac{1}{2x_q} - \frac{1}{2x_d{'}}\right)\times$$

$$\times E_m e^{-\frac{t}{T_d{'}}}\cos[(\omega_к + 1){+}\theta_0] + \left(\frac{1}{2x_q} + \frac{1}{2x_d{'}}\right)\times$$

$$\times E_m e^{-\frac{t}{T_d{'}}}\cos[(\omega_к - 1)t - \theta_0]\,. \qquad (24.56)$$

In this equation, the values T_d ', T_a' and ω_k are determined, respectively, by formulas (24.50), (24.33) and (24.32).

The accuracy of expressions for individual members in equation (24.56) is uneven. The coefficients in round brackets in the last two terms were determined in the assumption r = 0. Therefore, at the initial moment, the short circuit current while calculating it by (24.56) will be equal to zero only if in all terms we put r = 0. Then, formulas (24.56) and (24.32) will take the following form

$$i_к = -\frac{E_m}{x_d}\cos(t+\theta_0) - \left(\frac{1}{x_d{'}} - \frac{1}{x_d}\right)E_m e^{-\frac{t}{T_d{'}}}\cos(t+\theta_0) -$$

$$-\left(\frac{1}{2x_q} - \frac{1}{2x_d{'}}\right)E_m e^{-\frac{t}{T_a{'}}}\cos(2t+\theta_0) + \left(\frac{1}{2x_q} - \frac{1}{2x_d{'}}\right)E_m e^{-\frac{t}{T_a{'}}}\cos\theta_0, \quad (24.57)$$

$$\omega_к = 1. \quad (24.58)$$

Equation for the excitation winding current

We find an equation for the excitation current at sudden short circuit of the synchronous machine without a damer winding at $n = const$ i $U_f = const$.

To simplify the solution, we accept $r_s = 0$.

Substituting in (62,119) the expression for $i_d(p) = \Delta i_d(p)$ with (24.49) and $\Delta u_f(p) = 0$, we obtain for the free current in the excitation winding the equation:

$$\Delta i_f(p) = E_m \frac{x_{fa}}{x_f x_d{'}} * \frac{p}{p^3+\alpha_d{'}p^2+p+\alpha_d{'}}. \quad (24.59)$$

The denominator of the second fraction in the last expression can be represented as a cubic equation (24.51), whose roots are determined by (24.52). Expanding the operator expression (24.59) under the assumptions (24.53), we deduce

$$\Delta i_f = \frac{E_m}{x_d{'}} * \frac{x_{fa}}{x_f} e^{-\frac{t}{T_d{'}}} - \frac{E_m}{x_d{'}} * \frac{x_{fa}}{x_f}\cos(t+\varphi_f), \quad (24.60)$$

where

$$tg\ \varphi_f = \alpha_d{'}. \quad (24.61)$$

The second periodic component of the free current in the excitation winding is induced by the flow generated by the aperiodic currents in the stator. In a real machine, these currents, the flow, and the corresponding component of the excitation current are damped with the time constant T_a. Taking into account this and (24.34), equation (24.60) takes the following form:

$$\Delta i_f = I_m{'}\frac{x_{fa}}{x_f} e^{-\frac{t}{T_d{'}}} - I_m{'}\frac{x_{fa}}{x_f} e^{-\frac{t}{T_a{'}}}\cos(t+\varphi_f). \quad (24.62)$$

ull current of the excitation winding at short circuit

$$i_f = I_f + \Delta i_f = I_f + I_m{'}\frac{x_{fa}}{x_f} e^{-\frac{t}{T_d{'}}} - I_m{'}\frac{x_{fa}}{x_f} e^{-\frac{t}{T_a{'}}}\cos(t+\varphi_f); \quad (24.63)$$

it consists of a steady current of excitation i_f, aperiodic current, fading with the time constant T_d' and a periodic current fading with the time constant T_a'.

2.24. Equation for short-circuit currents of a synchronous machne with a damper winding

Equation for $r_{1d} \neq 0, \quad r_{1q} \neq 0, \quad r = r_f = 0\,,\ n = const\ i\ U_f = const$

At short circuit o the synchronous machine with a damper winding in the expression for the short circuit current a new, so-called, super-transitive component appears (in addition to the constituent current of the machine without a damper winding). In determining the latter for simplification, we consider

$$r = 0, \ \ r_f = 0. \tag{25.1}$$

Then formula (66, 17a) will have the following form

$$x_d(p) = \frac{x_d{'} + p x_d{''} T''_{d0}}{1 + pT''_{d0}}, \tag{25.2}$$

and formula (66.34) for x_q (p) remains unchanged.

Substituting values $x_q(p),\ x_d(p)$ i r and *r* from (66,34), (25.2) and (66,42), we deduce:

$$Z_d{'} = \frac{p(x_d{'} + p x_d{''} T''_{d0})}{1 + pT''_{d0}},$$

$$Z_q{'} = \frac{p(x_q + p x_q{''} T_{1q})}{1 + pT_{1q}}. \tag{25.3}$$

Substituting the corresponding statements from (25.2), (66.34) and (25.3) into (23.3) and taking into account the lack of current at the initial moment, we deduce:

$$\Delta i_d(p) = i_q(p) = -\frac{1 + T''_{d0}}{(1 + p^2)(x_d{'} + p x_d{''} T''_{d0})} E_m\,,$$

$$\Delta i_q(p) = i_d(p) = -\frac{p(1 + pT_{1q})}{(1 + p^2)\left(x_q{'} + p x_q{''} T_{1q}\right)} E_m \tag{25.4}$$

The roots of the equations composed of denominators of the last expressions are:

$$p_{1d} = -\frac{x_d{'}}{x_d{''} T''_{d0}} = -\frac{x_d{'} r_{1d}}{x_d{''} x'_{1d}} = -\frac{1}{T_d{''}} = -\alpha_d{''},$$

$$p_2 = j, \qquad p_3 = -j \tag{25.5}$$

$$p_{1q} = -\frac{x_q}{x_q{''} T_{1q}} = -\frac{1}{T_q{''}} = -\alpha_q{''},$$

$$p_2 = j, \qquad p_3 = -j \tag{25.6}$$

$$1 + \alpha''^2_d \approx 1 + \frac{1}{T''^2_d} = 1 \ ,$$

$$1 + T''^2_{d0} \approx T''_{d0}, \tag{25.7}$$

From (25.4) - (25.7) we find the following original expressions for currents:

$$i_d = -E_m \left[\frac{1}{x_d'} + \left(\frac{1}{x_d''} - \frac{1}{x_d'}\right) e^{-\frac{t}{T_d''}} - \frac{1}{x_d''} \cos\,(t + \varphi_d\right],$$

$$i_q = -E_m \left[\frac{x_q'' - x_q}{x_{1q} x''^2_q} r_{1q} e^{-\frac{t}{T_q''}} - \frac{1}{x_q''} \cos\,(t + \varphi_q\right], \tag{25.8}$$

where

$$tg\varphi_d = \frac{T''_{d0} T_d'' - 1}{T_d'' + T''_{d0}},$$

$$tg\varphi_q = \frac{1 + T_{1q} T_q''}{T_q'' - T_{1q}} \ . \tag{25.9}$$

Substituting the expressions for i_d i i_q from (25.8) in (23.7), we obtain for the short circuit current in phase *a*

$$i_к'' = -E_m \left\{\frac{1}{x_d'} \cos(t + \theta_0) + \left[\left(\frac{1}{x_d''} - \frac{1}{x_d'}\right) e^{-\frac{t}{T_d''}} \cos(t + +\theta_0) - \right.\right.$$

$$\left.- \frac{x_q'' - x_q}{x_{1q} x''^2_q} r_{1q} e^{-\frac{t}{T_q''}} \sin(t + \theta_0)\right] - \left[\frac{1}{2x_d''} \cos(\varphi_d - -\theta_0) - \frac{1}{2x_q''} \sin(\varphi_d - \theta_0)\right] -$$

$$\left. - \left[\frac{1}{2x_d''} \cos(2t + \varphi_d + \theta_0) - - \frac{1}{2x_q''} \sin\big(2t + \varphi_q + \theta_0\big)\right]\right\} \tag{25.10}$$

The resulting expression for the short circuit current includes the following four components.

a) Transient symmetrical undisturbed current

$$i_к' = -\frac{E_m}{x_d'} (t + \theta_0). \tag{25.11}$$

In a real machine, this current decays with the time constant T_d' up to the value of the steady-state short circuit current

b) Supertransitional symmetric component, fading with the time constant

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$$T_d'' = \frac{x_d''}{x_d'} T''_{d0} \,,$$
$$T_q'' = \frac{x_q''}{x_q'} T_{1q}, \qquad (25.12)$$

$$\Delta i_{к}'' = -\left[\left(\frac{1}{x_d''} - \frac{1}{x_d'}\right) e^{-\frac{t}{T_d''}} \cos(t + \theta_0) - \frac{x_q'' - x_q}{x_{1q} x''^2_q} r_{1q} e^{-\frac{t}{T_q''}} sin(t + \theta_0)\right] E_m. \quad (25.13)$$

The second term in square brackets is small compared to the first one; neglecting them, we will write

$$\Delta i_{к}'' \approx -\left(\frac{1}{x_d''} - \frac{1}{x_d'}\right) E_m e^{-\frac{t}{T_d''}} \cos(t + \theta_0). \quad (25.14)$$

c) Aperiodic current

$$i''_{кa} = \left[\frac{1}{2x_d''} \cos(\varphi_d - \theta_0) - \frac{1}{2x_q''} \sin\left(\varphi_q - \theta_0\right) E_m.\right] \quad (25.15)$$

In a real cmachine, this current decays with the time constant

d) Symmetrical double-frequency current

$$i''_{кa} = \left[\frac{1}{2x_d''} \cos(2t + \varphi_d + \theta_0) - \frac{1}{2x_q''} \sin\left(2t + \varphi_q + \theta_0\right) E_m.\right] \quad (25.16)$$

In a real machine, the dual frequency current fades with the time constant T_a.

Equation with $r \neq 0, \quad r_f = r_{1d} = r_{1q} = 0 \,, U_f = const и \;\; n = const$

To determine the time constant of short circuit currents attenuation of aperiodic and dual frequencies we find the equation for short circuit current at

$$r_f = r_{1d} = x_{1q} = 0$$
$$T_{d0} = T_{1d} = T_{1q} = \infty \qquad (25.17)$$

in this case formulas (66.17a) and (66.34) will take the form:

$$x_d(p) = x_d'' \,,$$
$$x_q(p) = x_q''. \qquad (25.18)$$

Substituting the values x_d (p) and x_q (p) from (25.18) into (23.6), we deduce:

$$i_d(p) = -\frac{1}{p^2 + 2\alpha_a'' p + d} \frac{E_m}{x_d''} \,, \qquad (25.19)$$

$$i_q(p) = -\frac{r + px_d''}{p^2 + 2\alpha_a''p + d}\frac{E_m}{x_d''x_q''} \,,$$

where

$$2\alpha_a'' = \frac{x_q'' + x_d''}{x_q''x_d''}r = 2\frac{1}{T_a''}\,,$$

$$d = 1 + \frac{r^2}{x_q''x_d''}\,. \tag{25.20}$$

The roots of the equation

$$N(p) = p^2 + 2\alpha_a''p + d = 0, \tag{25.21}$$

will be

$$p_{1,2} = -\alpha_a'' \pm \sqrt{\alpha''^2_a - d} = -\alpha_a'' \pm j\omega_к'' \tag{25.22}$$

or, taking into account (25.20)

$$p_{1,2} = -\frac{x''+x_d''}{2x_q''x_d''}r \pm j\sqrt{1 - \left(\frac{x_d''-x_q''}{2x_q''x_d''}r\right)^2} \,, \tag{25.23}$$

where

$$\omega_к'' = \sqrt{1 - \left(\frac{x_d''-x_q''}{2x_q''x_d''}r\right)^2} \,. \tag{25.24}$$

The original currents will take the form of formulas (24.25)

To simplify the calculations, we neglect the influence of the active resistance of the stator r on the values of coefficients $A_d,\ B_d, A_q,$ i B_q, that is, in determining these coefficients we will assume $r = 0;\ \ \alpha_a'' = 0;\ \ p_1 = j,\ \ \ p_2 = -j$.

At the same time, the originals of short circui currents will have the following form:

$$i_d = \left[-\frac{x_q''}{r^2 + x_q''x_d''} + \frac{1}{x_d''}e^{-\alpha_a''t}\cos\omega_к t\right]E_m \,,$$

$$i_q = \left[-\frac{r}{r^2 + x_q''x_d''} - \frac{1}{x_q''}e^{-\alpha_a''t}\,sin\,\omega_к t\right]E_m. \tag{25.25}$$

Substituting the expressions for i_d and i_q from (25.25) into (23.7), we obtain for the instantaneous value of the short circuit current in phase *a*

$$i''_{ка} = -\left[x_q''\cos(t + \theta_0) - r sin(t + +\theta_0)\right]\frac{E_m}{r^2 + x_q''x_d''} +$$

$$+\left(\frac{1}{2x_d{}''}+\frac{1}{2x_q{}''}\right)E_m e^{-\frac{t}{T_a{}''}} cos[(\omega_к - -1)t - \theta_0] +$$
$$+\left(\frac{1}{2x_d{}''}-\frac{1}{2x_q{}''}\right)E_m e^{-\frac{t}{T_a{}''}} cos[(\omega_к + 1)t + \theta_0]. \qquad (25.26)$$

Thus, the short circuit current of the synchronous machine in which the active rsistances of the excitation and damper windings are equal to zero, with the above assumptions, has the following components.

a) Symmetric undamped short circuit current of steady-state synchronous frequency mode equal to

$$-x_q{}''[\cos(t+\theta_0) - r\sin(t+\theta_0)]\frac{E_m}{r^2+x_q{}''x_d{}''}. \qquad (25.27)$$

b) The current determined by the formula

$$\left(\frac{1}{2x_d{}''}+\frac{1}{2x_q{}''}\right)E_m e^{-\frac{t}{T_a{}''}} \cos[(\omega_к - 1)t + \theta_0], \qquad (25.28)$$

which attenuates with the time constant [see (25.20)]

$$T_a{}'' = \frac{1}{\alpha_a{}''} = \frac{2x_q{}''x_d{}''}{(x_q{}'+x_d{}'')r} \qquad (25.29)$$

and changes with a very small frequency, which according to (25.20) is equal to:

$$\omega''_{ка} = (\omega_к{}'' - 1)\omega_c = \left[\sqrt{1-\left(\frac{x_d{}''-x_q{}''}{2x_q{}''x_d{}''}r\right)^2} - 1\right]\omega_c \approx 0. \qquad (25.30)$$

c) Symmetrical, fading with the time constant T_a " current of practically double frequency

$$\left(\frac{1}{2x_d{}''}-\frac{1}{2x_q{}''}\right)E_m e^{-\frac{t}{T_a{}''}} \cos[(\omega_к + 1)t + \theta_0]. \qquad (25.31)$$

Full expression for a sudden three-phase short circuit current

From the above cases, we can conclude that the current of a sudden three-phase oscillation of the real synchronous cmachine with a damper winding has the following components:

a) fixed symmetrical non-damped short circuit current of synchronous frequency (24.8);

b) transitive symmetric component of synchronous frequency, attenuating with the time constant T_d '[see Fig. (24.43), (24.50)];

c) The transient symmetric component of synchronous frequency, decreasing with the time constant T_d " and T_q " [see Fig. (25.12), (24.13)];

d) practically aperiodic short circuit current, which can be considered as damped with the time constant T_a " [see Fig. (25. 28), (25.29)];

e) the symmetric current is practically of double frequency [see (25.31)].

Thus, the full short circuit current in phase *a* is equal to

$$i_{\text{ка}} = -\left[x_q \cos(t+\theta_0) - r\sin(t+\theta_0)\right]\frac{E_m}{r^2 + x_q x_d} - \left[x_q \cos(t+\theta_0) - r\sin(t+\right.$$

$$\left.+\theta_0)\right] * \frac{x_q(x_d - x_d{'})E_m}{(r^2 + x_q x_d{'})(r^2 + x_q x_d)} e^{-\frac{t}{T_a{'}}} - \left[\left(\frac{1}{x_d{''}} - \frac{1}{x_d{'}}\right) E_m e^{-\frac{t}{T_d{''}}} \cos(t+\theta_0) - \right.$$

$$\left. - \frac{x_q{''} - x_q}{x_{1q} x''^2_q} r_{1q} E_m e^{-\frac{t}{T_q{''}}} \sin(t+\theta_0)\right] + \left(\frac{1}{2x_d{''}} + \frac{1}{2x_q{''}}\right) E_m e^{-\frac{t}{T_a{''}}} \cos[(\omega_{\text{к}} - 1)t - $$

$$-\theta_0] + \left(\frac{1}{2x_d{''}} - -\frac{1}{2x_q{''}}\right) E_m e^{-\frac{t}{T_a{''}}} cos[(\omega_{\text{к}} + 1)t + \theta_0]. \qquad (25.32)$$

If we assume the active resistances of windings to be equal to zero in all terms and factors, except for the time constant, then the last equation will take the following form:

$$i_{\text{ка}} = -E_m\left[\frac{1}{x_d} + \left(\frac{1}{x_d{'}} - \frac{1}{x_d}\right) e^{-\frac{t}{T_d{'}}} + \left(\frac{1}{x_d{''}} - \frac{1}{x_d{'}}\right) e^{-\frac{t}{T_d{'}}}\right] \cos(t+\theta_0) + \left(\frac{1}{2x_d{''}} + \right.$$

$$\left. + \frac{1}{2x_q{''}}\right) E_m e^{-\frac{t}{T_a{''}}} \cos\theta_0 + \left(\frac{1}{2x_d{''}} - \frac{1}{2x_q{''}}\right) E_m e^{-\frac{t}{T_a{''}}} \cos(2t+\theta_0). \quad (25.33)$$

Equations and the time constant of currents in excition an damper windings

Let's find the operator equations for currents in excitation and damper windings at a sudden short circuit of the synchronous machine at no-load operation and determine the time constant therefrom, with which the free currents in these windings attenuate.

We will continue to consider for all the time of short circuit the rotational speed and the excitation voltage to be constant. In this case, $\Delta U_f = 0$,, and we write the formulas in the following form:

$$\Delta i_f(p) = \frac{p^2(x_{f1}x_{1ad} - x_{fa}x_{1d}) - p x_{fa} r_{1d}}{A(p)} \Delta i_d(p), \qquad (25.34)$$

$$\Delta i_{1d}(p) = \frac{p^2(x_{fa}x_{1f} - x_{1ad}x_f) - p x_{1ad} r_f}{A(p)} \Delta i_d(p),$$

$$\Delta i_{1q}(p) = -\frac{p x_{1aq}}{r_{1q} + p x_{1q}} \Delta i_q(p), \qquad (25.35)$$

To simplify, we assume that r = 0; then equations (23.6) will take the following form [53]:

$$i_d(p) = -\frac{E_m}{(1+p^2)x_d(p)}$$

$$i_q(p) = -\frac{pE_m}{(1+p^2)x_q(p)} \tag{25.36}$$

Substituting in (25.36) the values $X_d(p)$ and $X_q(p)$ from (66,17a) and (66. 34), and the expressions obtained after this into (25.34) and (25.35), we have:

$$\Delta i_f(p) = -\frac{p^2(x_{f1}x_{1ad}-x_{fa}x_{1d})-px_{fa}r_{1d}}{Ш_{f1}}E_m\,, \tag{25.37}$$

$$\Delta i_{1d}(p) = -\frac{p^2(x_{fa}x_{1f}-x_{1ad}x_f)}{Ш_{f1}}E_m\,, \tag{25.38}$$

where

$$Ш_{f1} = (1+p^2)\{x_f x'_{1d}x_d''p^2 + [x_d(x_f x_{1d}+x_{1d}r_f) - x_{ad}(x_{fм}r_{1d} +$$

$$+x_{1мd}r_f)]p + x_d r_f r_{1d}\}.$$

$$\Delta i_{1q}(p) = i_{1d}(p) = -\frac{p^2 x_{1aq}E_m}{(1+p^2)(x_q+px_q''T_{1q})}\,. \tag{25.39}$$

The solution of the equations obtained and the determination of the time constant of free currents in the excitation and damper windings can be greatly simplified. To do this, when calculating the free current in the excitation winding and its transient time constant, T_d' should be neglected by the currents in the damper winding, assuming they are rapidly damped, that is, assuming $r_{1d} = \infty$, and when calculating the current in the damper winding and its super-transient time constant T_d'' consider the free current in the excitation winding non-damping, that is $r_f = 0$. These assumptions will not cause big errors, since in most synchronous machines, the super-transient time constant T_d'' is small in comparison with the transient time constant T_d'.

Assuming in (25.37) r_1d = ∞, and in (25.38) r_f = 0, we obtain

$$\Delta i_f(p) = \frac{p}{(1+p^2)[p(x_d x_f - x_{ad}x_{fм})+x_d r_f]}x_{fa}E_m,$$

$$\Delta i_d(p) = -\frac{p(x_{fa}x_{1f}-x_{1ad}x_f)}{(1+p^2)[px_f x'_{1d}x_d'' + (x_d x_f - x_{ad}x_{fм})r_{1d}]}E_m. \tag{25.40}$$

Then, using (28,4) and (62,15), we have

$$\Delta i_f(p) = \frac{E_m}{x_d'}\frac{x_{fa}}{x_f}\frac{p}{(1+p^2)\left(p+\frac{x_d}{x_d'T_{d0}}\right)}, \tag{25.41}$$

or

$$\Delta i_f(p) = \frac{E_m}{x_d'}\frac{x_{fa}}{x_f}\frac{p}{p^3+\alpha_d' p^2+p+\alpha_d'}, \qquad (25.42)$$

where α_d 'is determined by (24.50).

Comparing (25.42) with (24.59), we come to the conclusion that under assumptions made, the excitation current is determined by one and the same equation.

This result could have been anticipated in advance, since putting $r_{1d} = \infty$, we thus would remove the damper winding from the machine.

From (25.40) and (28.4) we find

$$i_{1d}(p) = \frac{E_m}{x_d'}\frac{x_{1ad}x_f - x_{fa}x_{1f}}{x_f x'_{1d}}\frac{p}{p^3+\alpha_d'' p^2+p+\alpha_d''}, \qquad (25.43)$$

where α_d'' is determined by (25.5)

Comparing the obtained equation with (24.59), we write for the origin of the current in a damper winding on the longitudinal axis the expression similar to (24.60)

$$i_{1d}(p) = \frac{E_m}{x_d'}\frac{x_{1ad}x_f - x_{fa}x_{1f}}{x_f x'_{1d}}\frac{p}{p^3+\alpha_d'' p^2+p+\alpha_d''}, \qquad (25.43)$$

where $I_m' = \frac{E_m}{x_d''}$, $tg\varphi_{1d} = \alpha_d''$.

The roots of the equation, compiled from the denominator of equation (25.39), are equal to

$$p_1 = -\alpha_q'' = -\frac{1}{T''_{1q}} = -\frac{x_q}{x_q'' T_{1q}},$$

$$p_2 = +j, \quad p_3 = -j\,.$$

Taking advantage of formula (28), given in appendix 6, for expanding the operative expression (25.39), we will put the following

$$i_{1q} = \frac{x_{1aq}}{x_{1q}}\frac{E_m}{x_q''}\left[-\frac{\alpha_q''}{1+\alpha''^2_q}e^{-\frac{t}{T''_{1q}}} + \frac{1}{\sqrt{1+\alpha''^2_q}}\cos(t+\varphi_{1q})\right]. \qquad (25.45)$$

Assuming $1+\alpha''^2_q + 1$ and taking into account the attenuation of the aperiodic short-circuit current, we have

$$\boldsymbol{i_{1q}} = \frac{x_{1aq}}{x_{1q}}\frac{E_m}{x_q''}\left[-\boldsymbol{\alpha_q'' e^{-\frac{t}{T''_{1q}}} = e^{-\frac{t}{T_a''}}\cos(t+\varphi_{1q})}\right], \qquad (25.46)$$

where

$$tg\,\psi_{1q} = -T_q''\,. \qquad (25.47)$$

2.25. Changes in the voltage of a synchronous generator without a damper winding under sudden load on

Approximate solutions (without taking into account the active resistance of the stator winding and load)

At sudden load on, the voltage of synchronous generator is subject to a rather sharp change, which can adversely affect consumers. To determine the voltage change at load on, we make the following assumptions:

1) rotor speed is constant and equals ω_r;

2) the load on occurred at no-load operation of the generator, i.e. $I_d(0) = I_q(0) = 0$;

3) the active resistances of the stator winding and cos φ≈0.9÷0.8 are neglected (26.1);

4) the inductive load resistance x_L is staedy;

5) transformer E. M. F. in the stator is neglected, that is, we suppose that

$$\frac{d\psi_d}{dt} = \frac{d\psi_q}{dt} = 0 \tag{26.1}$$

Under the above conditions and assumptions, the respective positions and equalities will take place.

At the initial moment, the vector diagram has the form shown in Fig. 23.1. At this

$$u_d(0) = 0; \quad u_q(0) = E_m = U_{m0} = x_{af} I_f = x_{af} \frac{U_f}{r_f}; \tag{26.2}$$

Equation (15. 15) will have the following form

$$u_d = -\omega_r \psi_q$$
$$u_q = \omega_r \psi_q \tag{26.3}$$

the load current will be purely reactive, i.e. short circuit

$$i_q = 0, \quad i_d = -i_m, \tag{26.4}$$

where i_m is the current amplitude.

Since there is no damper winding, then

$$\psi_q \approx x_q i_q \approx 0 \tag{26.5}$$

and, according to (26.3) $u_d = 0$.

Thus,

$$u_q = u_m = \omega_r \psi_d \, , \tag{26.6}$$

where u_m is the voltage amplitude of the network.

The voltage drop in the inductive load resistance can be determined by (87. 14) by putting

$$i_x(0) = i_y(0) = 0,$$
$$\omega_{zx} = \omega_{ad} = \omega_r. \tag{26.7}$$

If the values $px_L i_x(p)$ and $px_L i_y(p)$ are neglected comparing with $\omega_{zx} i_y(p)$ and $\omega_{zx} i_x(p)$, then in the axes *d, q* the system (27. 14) will take the following form:

$$u_{Ld}(p) = 0,$$
$$u_{Ld}(p) = \omega_r x_L x i_d(p). \tag{26.8}$$

The voltages on the clamps of the machine and the network are equal, but opposite in the direction, that is

$$u_q = -u_{Ld}\ , \tag{26.9}$$

or according to (26.6) and (26.8)

$$u_m = u_q = \omega_r \psi_d = -\omega_r x_L i_d; \tag{26.10}$$

therefore

$$\psi_d = -x_L i_d. \tag{26.11}$$

Equation (62.90) under conditions of this problem will have the following form

$$-x_L i_d(p) = G_{cd} u_f(p) + X_d(p) i_d(p) + pG_{cd}\psi_f(0); \tag{26.12}$$

From here

$$i_d(p) = -\frac{G_{cd}u_f(p)}{x_L + X_d(p)} - \frac{pG_{cd}\psi_f(0)}{x_l + X_d(p)} = i_{d1}(p) + i_{d0}(p), \tag{26.13}$$

where

$$i_{d1}(p) = -\frac{G_{cd}u_f(p)}{x_L + X_d(p)} = -\frac{x_{af}u_f(p)}{r_f(x_l + x_d')T_{d0}\left(p + \frac{1}{T'_{d0}}\right)}, \tag{26.14}$$

$\psi_f(0)$ is the initial flux linkage of the excitation winding that is equal to
In accordance with (62. 18), (62. 35) and (26.2)

$$i_{d0}(p) = -\frac{pG_{cd}\psi_f(0)}{x_L + X_d(p)} = -\frac{U_{m0}}{x_L + x_d'}\frac{p}{p + \frac{1}{T'_{dL}}}, \tag{26.16}$$

where

$$T'_{dL} = \frac{x_L + x_d'}{x_L + x_d} T_{d0}. \tag{26.17}$$

From (26.16) we have

$$i_{d0} = -\frac{U_{m0}}{x_L + x_d'} e^{-\frac{t}{T'_{dL}}}. \tag{26.18}$$

In the absence of the excitation regulator and the constant excitation voltage U_f, the formula (26.14) will have the following form

$$i_{d1}(p) = -\frac{U_{m0}}{(x_L+x_d{'})T_{d0}} \frac{1}{p+\frac{1}{T'_{dL}}} \ , \tag{26.19}$$

From here

$$i_{d1} = -\frac{U_{m0}}{x_L+x_d}\left(1 - e^{-\frac{t}{T'_{dL}}}\right). \tag{26.20}$$

From (26.13), (26.16) and (26.20) for the stator current we find the expression

$$i_d = i_{d1} + i_{d0} = -U_{m0}\left[\frac{1}{x_L+x_d} + \frac{x_d-x_d{'}}{(x_L+x_d)(x_L+x_d{'})} e^{-\frac{t}{T'_{dL}}}\right]. \tag{26.21}$$

Substituting the last expression for i_d into (26.10) with $\omega_r = 1$ gives for the network voltage amplitude the equation:

$$u_{m1} = -x_L i_d = U_{m0}\left[\frac{x_L}{x_L+x_d} + \frac{(x_d-x_d{'})x_L}{(x_L+x_d)(x_L+x_d{'})} e^{-\frac{t}{T'_{dL}}}\right]. \tag{26.22}$$

The network voltage in this case varies by curve 1. 26.2. 3 (26.22) at t = 0 and $t = \infty$ we deduce:

$$U_{t0} = \frac{x_l}{x_L + x_d{'}} U_{m0} \ ,$$
$$U = \frac{x_l}{x_L + x_d} U_{m0}. \tag{26.23}$$

As follows from the last formulas and 26.2, an instantaneous voltage drop occurs at the initial moment of load on

$$\Delta U_{t0} = \frac{x_d{'}}{x_l+x_d{'}} U_{m0} \ . \tag{26.24}$$

Subsequently, the voltage continues to decrease with the time constant T'_{dL} to a constant value equal to U_∞

The maximum voltage drop will then be equal to

$$\Delta U_{max} = \frac{x_d}{x_L+x_d} U_{m0} \ . \tag{26.25}$$

The regulator increases the excitation voltage according to the rectiliniar law

Let the excitation controller increases the excitation voltage at the load on by the following law

$$u_f = U_{f0} + \Delta u_f = U_{f0} + U_{f0} k_{\text{в}} t \ , \tag{26.26}$$

where $k_{\text{в}}$ is the constant in magnitude velocity of the excitation voltage increase. Let’s rewrite the last equation in the operator form:

$$u_f = (p) = U_{f0} + \frac{k_{\text{в}} U_{f0}}{p}. \tag{26.27}$$

Substituting this value $u_f(p)$ into (26.12), we obtain for the stator current in addition to the components $i_{\text{d}0}$ and $i_{\text{d}1}$ the third component $i_{\text{d}2}$, whose operator expression has the following form

$$i_{d2}(p) = \frac{k_{\text{в}} G_{cd} U_{f0}}{[x_L + X_d(p)]p} = \frac{k_{\text{в}} x_{af} U_{f0}}{(x_L + x_d')\left(p + \frac{1}{T'_{dL}}\right) p x_f}\ , \tag{26.28}$$

or

$$p i_{d2}(p) = \frac{k_{\text{в}} x_{af} U_{f0}}{(x_L + x_d') x_f} \frac{1}{p + \frac{1}{T'_{dL}}} = \frac{d}{dt} i_{d2}\ ,$$

From here

$$\frac{d i_{d2}}{dt} = \frac{k_{\text{в}} x_{af} T'_{dL} U_{f0}}{(x_L + x_d') x_f} \left(1 - e^{-\frac{t}{T'_{dL}}}\right). \tag{26.29}$$

Integrating the last equation, we deduce

$$i_{d2} = \frac{k_{\text{в}} U_{m0}}{x_L + x_d} \left[t - T'_{dL}\left(1 - e^{-\frac{t}{T'_{dL}}}\right)\right]. \tag{26.30}$$

The voltage of the network is found from the equation

$$u_m = x_L \ell_r (i_{d0} + i_{d1} + i_{d2}) = (u_{m1} + x_L i_{d2}) \ell_r. \tag{26.31}$$

$$u_m = U_{m0}\left\{K_1 + K_2 e^{-\frac{t}{T'_{dL}}} + K_1 k_{\text{в}}\left[t - T'_{dL}\left(1 - e^{-\frac{t}{T'_{dL}}}\right)\right]\right\}, \tag{26.32}$$

where

$$K_1 = \frac{x_l}{x_L + x_d}\ ,$$

$$K_2 = \frac{(x_d - x_d') x_L}{(x_L + x_d)(x_L + x_d')}. \tag{26.33}$$

If the controller operates only after the time t_1 of the radian after the load on, then in the last right-hand member (26.32), taking into account the presence of the controller, *t* should be replaced by $t - t_1$;
Then

$$u_m = U_{m0}\left\{K_1 + K_2 e^{-\frac{t}{T'_{dL}}+} + K_1 k_{\text{в}}\left[t - t_1 - T'_{dL}\left(1 - e^{-\frac{t-t_1}{T'_{dL}}}\right)\right]\right\}. \tag{26.34}$$

The last equation completely coincides with the Andersen's formula derived by another way.

When using this equation, one should take *t*-t_1 $= 0$ in the time interval from 0 to t_1.

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In. Fig. 26.2 the curve 3 corresponds to the equation (26. 32), and the curve 2 is the voltage component due to the controller action.

As can be seen from curve 3, the voltage u_m at the beginning decreases, at the time t_m the latter reaches the minimum value, and then begins to increase infinitely. The latter is due to the fact that when the equations were deduced, we neglected the saturation and adopted the law of increasing the excitation voltage as rectilinear. However, this assumption has little effect on the magnitude of maximum voltage failure ΔU_{max} of its onset t_m.

To find t_m and ΔU_{max}, we take the derivative from u_m (26.34), equate it with zero and solve the equation derived with respect to time [54].

Where 26.2

$$t_m = T'_{dL} \ln\left[\frac{x_d - x_d'}{(x_L + x_d')k_{\text{в}}T'_{dL}} + e^{-\frac{t_1}{T'_{dL}}}\right]. \qquad (26.35)$$

Substitution of the expression for t_m into (26.34) enables to define $U_{min}$and gives the following expression for the maximum voltage failure in relative units:

$$\Delta U_{*max} = 1 - U_{*min} = 1 -$$

$$-\frac{x_L}{x_L + x_d}\left\{k_{\text{в}}\left[T'_{dL}\ln\left(\frac{x_d - x_d'}{(x_L + x_d')k_{\text{в}}T'_{dL}} + e^{-\frac{t}{T'_{dL}}}\right) - -t_1\right] + 1\right\} \qquad (26.36)$$

In self-excited synchronous generators, the excitation voltage usually has two components, one of which can be taken directly proportional to the voltage, and the other one as the generator current, that is

$$u_f = k_u u_m + k_i i_m\,, \qquad (26.37)$$

where u_m, i_m are the amplitudes of voltage and current;

k_u, k_i are coefficients of proportionality.

If neglecting the active oscillations of both the generator and the load as well as the transformer E. M. F. in the generator and the load, then, as shown above

$$u_m = u_q, \quad i_m = -i_d. \qquad (26.38)$$

At the same time, considering (26.10),

$$u_f = k_u u_q - k_i i_d = -(\nu_r x_L k_u + k_i) i_d = -k_{ui} i_d, \qquad (26.39)$$

where

$$k_{ui} = k_i + x_L k_u \nu_r. \qquad (26.40)$$

At the initial moment of idling

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$$u_f(0) = k_u U_{m0} = U_{f0}, \tag{26.41}$$

where

$$k_u = \frac{U_{f0}}{U_{m0}}. \tag{26.42}$$

The coefficient $k_i$can be determined from the condition of obtaining a given, for example, nominal voltage of the network $U_{m\text{н}}$ at given nominal current $i_{m\text{н}}$ and $cos\varphi$. Then from (26.37)

$$k_i = \frac{U_{f\text{н}} - k_u U_{m\text{н}}}{I_{m\text{н}}}, \tag{26.43}$$

where $U_{f\text{н}}$ is the excitation voltage, which provides the nominal stator voltage at the current of $I_{m\text{н}}$.

Substituting the expression for (26.39) into (26.12), we obtain$-x_L i_d(p) = -G_{cd} k_{ui} i_d(p) + X_d(p) i_d(p) + pG_{cd}\psi_f(0),$

where

$$i_d(p) = -\frac{pG_{cd}\psi_f(0)}{x_L + x_d(p) - k_{ui}G_{cd}}. \tag{26.44}$$

Substituting here the values $X_d(p)$ and $\psi_f(0)$ from (62. 18) and (26.15) and taking advantage of (62,15), we will deduce

$$i_d(p) = -\frac{pU_{m0}}{(x_L + x_d')\left(p + \frac{1}{T'_{ui}}\right)}, \tag{26.45}$$

where

$$T'_{ui} = \frac{x_L + x_d'}{x_L + x_d - \frac{r_{ui}x_{af}}{f}} T_{d0}. \tag{26.46}$$

Then

$$i_d = -\frac{U_{m0}}{x_L + x_d'} e^{-\frac{t}{T'_{ui}}}. \tag{26.47}$$

The equation for the network voltage will have the following form:

$$u_m = -x_L{\bullet}_r i_d = \frac{x_L{\bullet}_r}{x_L + x_d'} U_{m0} e^{-\frac{t}{T'_{ui}}}. \tag{26.48}$$

If $T'_{ui} > 1$, that is, if $\frac{k_{ui}x_{af}}{r_f} < x_L + x_d$, then $u_m$will change in curve 26.3. Reduced at the initial moment of switching on instantaneously by magnitude

$$\Delta U_{t0} = \frac{x_d'}{x_L + x_d'} U_{m0}, \tag{26.49}$$

then the voltage drops to zero with the time constant T_{ui}.

If $T'_{ui} < 1$, that is, if $\frac{k_{ui}x_{af}}{r_f} > x_L + x_d$, therefore, the generator has a fairly strong compounding, then the network voltage in the absence of saturation will change along curve 2, increasing infinitely. In the presence of saturation, the voltage in this case will increase in curve 3.

If

$$k_{ui} = (x_L + x_d)\frac{r_f}{x_{af}},$$

then

$$T'_{ul} = \infty \quad \text{i} \quad u_m = \frac{x_L}{x_L + x_d'} U_{m0}.$$

In this case, the voltage does not depend on the straight line 4.

Let's assume that in the compound synchronous generator, the excitation voltage consists of two components, one of which is constant and equal to U_{f0}, and the other, $k_i U i_m$, is directly proportional to the load current, that is,

$$u_f = U_{f0} + k_i i_m. \tag{26.50}$$

When there is a purely inductive set of load and we neglect the active resistance of the stator winding, the following equality is true (26.38).

Then

$$u_f = U_{f0} - k_i i_d. \tag{26.51}$$

The degree of compounding k can be determined by the following formula

$$k_i = \frac{U_{f\text{н}} - U_{f0}}{I_{m\text{н}}}, \tag{26.52}$$

where $U_{f\text{н}}$ is the excitation voltage corresponding to the given voltage of the network at the given load current $I_{m\text{н}}$.

Substituting u_f from (26.51) into (26.12), we deduce

$$-x_L i_d(p) = G_{cd}\left[U_{fo} - k_i i_d(p)\right] + X_d(p) i_d(p) + pG_{cd}\psi_f(0),$$

where

$$i_d(p) = -\frac{G_{cd}U_{f0} + pG_{cd}\psi_f(0)}{x_L + X_d(p) - k_i G_{cd}}. \tag{26.53}$$

Substituting here values $X_d(p)$ and ψ_f (0) from (26. 18) and (26.15), using (62. 15) we deduce

$$i_d(p) = -\frac{(1 + pT_{d0})U_{m0}}{(x_L + x_d')\left(p + \frac{1}{T_i'}\right)T_{d0}}, \tag{26.54}$$

where

$$T_i' = \frac{x_L + x_d'}{x_L + x_d \frac{k_i x_{af}}{r_f}} T_{d0}. \tag{26.55}$$

We find the load current equation by defining the original of equation (26.54), namely

$$i_d = -\frac{U_{m0}}{x_L + x_d \frac{k_i x_{af}}{r_f}} \left(1 - e^{-\frac{t}{T'_i}}\right) - \frac{U_{m0}}{x_L + x_d'} e^{-\frac{t}{T'_i}} \tag{26.56}$$

According to (26.10) and (26.56) at $\omega_r = 1$ is the network voltage

$$u_m = U_{m0} \left[\frac{x_L}{x_L + x_d \frac{k_i x_{af}}{r_f}} \left(1 - e^{-\frac{t}{T'_i}}\right) + \frac{x_L}{x_L + x_d'} e^{-\frac{t}{T'_i}} \right], \tag{26.57}$$

or

$$u_m = U_{m0} \left[\frac{x_L}{x_L + x_d \frac{k_i x_{af}}{r_f}} + \frac{x_L \left(x_d - \frac{k_i x_{af}}{r_f} \right) - x_d'}{(x_L + x_d')(x_L + x_d - \frac{k_i x_{af}}{r_f})} e^{-\frac{t}{T'_i}} \right]. \tag{26.58}$$

At t = 0

$$U_{t0} = \frac{x_L}{x_L + x_d'} U_{m0} \,. \tag{26.59}$$

Initial instantaneous voltage failure

$$\Delta U_{t0} = U_{m0} - U_{t0} = \frac{x_d'}{x_L + x_d'} U_{m0} \,. \tag{26.60}$$

If $x_d - \frac{k_i x_{af}}{r_f} = x_d'$, then the voltage will change in a straight line 1 26.4

If $x_L + x_d > \frac{k_i x_{af}}{r_f}$ i $x_d \frac{k_i x_{af}}{r_f} > x_d'$ the voltage changes along the curve 2 26.4

When $x_L + x_d > \frac{k_i x_{af}}{r_f}$, but $x_d - \frac{k_i x_{af}}{r_f} < x_d'$, the voltage will change along the curve 3 26.4.

If $x_L + x_d < \frac{k_i x_{af}}{r_f}$ and there is no saturation, then the voltage will grow along the curve 4 26.4.

The controller increases the excitation voltage according to the exponential function law.

Case 1. Let the excitation controller increase the excitation voltage at the inductive set of load according to the following law

$$u_f = U_{f0} + \Delta U_f \left(1 - e^{-\frac{t}{T'_{\text{в}}}}\right) = U_f - \Delta U_f e^{-\frac{t}{T'_{\text{в}}}}. \tag{26.61}$$

where

$$U_f = U_{f0} + \Delta U_f.$$

The same equation in operator form

$$u_f(p) = U_f - \Delta U_f \frac{p}{p+\frac{1}{T_B}}. \qquad (26.62)$$

Substituting this value $u_f(p)$ into (26. 12), we deduce

$$-x_L i_d(p) = G_{cd}\left[U_f - \Delta U_f \frac{p}{p+\frac{1}{T_B}}\right] + X_d(p)i_d(p) + pG_{cd}\Psi_f(0); \qquad (26.63)$$

From here

$$i_d(p) = i_{d0}(p) + i_{d1}(p) + i_{d2}(p), \qquad (26.64)$$

Where $i_{d0}(p)$ is determined by (26.16),

$$i_{d1}(p) = -\frac{G_{cd}U_f}{x_L+X_d(p)} = -\frac{E_m}{(x_L+x_d')\left(p+\frac{1}{T'_{dL}}\right)}, \qquad (26.65)$$

$$i_{d2}(p) = \frac{p\Delta u_f G_{cd}}{[x_L+X_d(p)]\left(p+\frac{1}{T_B}\right)} == \frac{p\Delta E_m}{\left(p+\frac{1}{T_B}\right)\left(p+\frac{1}{T'_{dL}}\right)(x_L+x_d')T_{d0}}, \qquad (26.66)$$

where

$$E_m = \frac{U_f}{r_f}x_{af} = I_f x_{af},$$

$$\Delta E_m = \frac{\Delta U_f}{r_f}x_{af} = \Delta I_f x_{af}. \qquad (26.67)$$

Expanding the operator expressions for currents, we obtain [55] for i_{d1} and i_{d0} expressions similar to (26.20) and (26.18), and for i_{d2}

$$i_{d2} = \frac{\Delta E_m T_B}{(x_L+x_d)(T_B-T'_{dL})}\left(e^{-\frac{t}{T_B}} - e^{-\frac{t}{T'_{dL}}}\right). \qquad (26.68)$$

Entire load current

$$i_d = -\frac{E_m}{x_L+x_d}\left(1-e^{-\frac{t}{T'_{dL}}}\right) - \frac{U_{m0}}{x_L+x_d'}e^{-\frac{t}{T'_{dL}}} +$$

$$+\frac{\Delta E_m T_B}{(x_L+x_d)(T_B-T'_{dL})}\left(e^{-\frac{t}{T_B}} - e^{-\frac{t}{T'_{dL}}}\right). \qquad (26.69)$$

Network voltage

$$u_m = \frac{x_L}{x_L+x_d}E_m + \left[\frac{x_L}{x_L+x_d'}U_{m0} - \frac{x_L}{x_L+x_d}E_m + \frac{x_L T_B \Delta E_m}{(x_L+x_d)(T_B-T'_{dL})}\right]e^{-\frac{t}{T'_{dL}}} -$$

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$$-\frac{x_L T_B \Delta E_m}{(x_L+x_d)(T_B-T'_{dL})} e^{-\frac{t}{T_B}} \tag{26.70}$$

or

$$u_m = \frac{U_{m0}}{x_L+x_d} + K_2 U_{m0} e^{-\frac{t}{T'_{dL}}} + \frac{\Delta E_m}{x_L+x_d}\left[1-\frac{T'_{dL}e^{-\frac{t}{T'_{dL}}}-T_B e^{-\frac{t}{T_B}}}{T'_{dL}-T_B}\right].$$

The time when the voltage takes the minimum value is the same

$$t_m = \frac{T_B T'_{dL}}{T'_{dL}-T_B}\left\{\frac{t_1}{T_B} - \ln\left[e^{-\frac{t}{T'_{dL}}} - -\frac{(x_d-x_d')U_{m0}}{(x_L+x_d')\Delta E_m}\left(\frac{T'_{dL}-T_B}{T'_{dL}}\right)\right]\right\}. \tag{26.71}$$

Case 2. Let the excitation controller increase the excitation voltage at purely inductive set of load by the respective law

The last equation in the operator form

$$u_f(p) = U_{f0}\frac{p}{p-\frac{1}{T_B}}. \tag{26.73}$$

Substitution of this value $u_f(p)$ into (26.12) gives

$$-x_L i_d(p) = G_{cd}U_{f0}\frac{p}{p-\frac{1}{T_B}} + X_d(p)i_d(p) + pG_{cd}\psi_f(0), \tag{26.74}$$

From here

$$i_d(p) = -\frac{pG_{cd}U_{f0}}{[x_L+x_d(p)]\left(p-\frac{1}{T_B}\right)} - \frac{pG_{cd}\psi_f(0)}{x_L+X_d(p)}, \tag{26.75}$$

or

$$i_d(p) = -\frac{pU_{m0}}{\left(p-\frac{1}{T_B}\right)\left(p+\frac{1}{T'_{dL}}\right)T_{d0}(x_L+x_d')} - \frac{pU_{m0}}{\left(p+\frac{1}{T'_{dL}}\right)(x_L+x_d')}. \tag{26.76}$$

Expanding this operator expression, we deduce

$$i_d = -U_{m0}\left[\frac{T_B}{(x_L+x_d)(T_B+T'_{dL})}\left(e^{\frac{t'}{T_B}} - e^{-\frac{t'}{T'_{dL}}}\right) + \frac{1}{x_L+x_d'}e^{-\frac{t}{T'_{dL}}}\right]. \tag{26.77}$$

Substituting this valuei_d into (26.10) at $\omega_r = 1$ for the network voltage gives

$$u_m = U_{m0}\left\{\frac{x_L T_B}{(x_L+x_d)(T_B+T'_{dL})}e^{\frac{t}{T_B}} + \left[\frac{x_L}{x_L+x_d'} - \frac{x_L T_B e^{-\frac{t}{T'_{dL}}}}{(x_L+x_d)(T_B+T'_{dL})}\right]e^{-\frac{t}{T'_{dL}}}\right\}. \tag{26.78}$$

Refined solutions (taking into account the active resistance of the stator winding and the load)

Unlike the previous cases, we will not neglect the influence of the active load resistances r_B and the stator winding r. We will keep considering as previously the set of load at idle operation and the rotor spinning speed to be constant [56].

To solve the problem in question, we will use equations (26. 37) to increase the magnitudes. Let us neglect the transformer E. M. F., that is, we will accept

$$p\Delta\psi_d(p) = p\Delta\psi_q(p) = 0\,.$$

Substituting an increase in the flux linkage from (66. 38) into (66. 37), we deduce:

$$\Delta u_d(p) = r\Delta i_d(p) - \omega_{ad}X_q(p)\Delta i_q(p),$$
$$\Delta u_q(p) = r\Delta i_q(p) + \omega_{ad}X_d(p)\Delta i_d(p) + \omega_{ad}G_{cd}\Delta u_f(p). \tag{26.79}$$

The voltage drop in the active $r_{\text{В}}$ and inductive x_L load resestances in accordance with (26. 25) under zero initial conditions in the axes *d, q* are equal to:

$$\boldsymbol{u_{zd}(p) = (r_{\text{В}} + px_L)i_d(p) - \omega_{ad}x_L i_q(p)},$$
$$u_{zq}(p) = (r_{\text{В}} + px_L)i_q(p) + \omega_{ad}x_L i_d(p). \tag{26.80}$$

Neglecting the transformer E. M. F., that is assuming

$$px_L i_d(p) \approx px_L i_q(p) \approx 0, \tag{26.81}$$

and replacing the magnitudes with their increments, we will rewrite the last equations in the following form:

$$\Delta u_{zd}(p) = r_{\text{В}}\Delta i_d(p) - \omega_{ad}x_L\Delta i_q(p),$$
$$\Delta u_{zq}(p) = r_{\text{В}}\Delta i_q(p) + \omega_{ad}x_L\Delta i_d(p) \tag{26.82}$$

The sum of the stator voltage increase and the voltage drop in the load is equal to

$$\Delta u_d(p) + \Delta u_{zd}(p) = r_r\Delta i_d - \omega_{ad}X_{qz}(p)\Delta i_q(p),$$
$$\Delta u_q(p) + \Delta u_{zq}(p) = r_r\Delta i_q - \omega_{ad}X_{dz}(p)\Delta i_d(p) + \tag{26.83}$$
$$+\omega_{ad}G_{cd}\Delta u_f(p),$$

where for machines without a damper winding

$$x_{zd} = x_L + x_d\,,$$
$$X_{zd}(p) = X_d(p) + x_L = \frac{x_L + x_d + p(x_L + x_d{}')T_{d0}}{1 + pT_{d0}}\,, \tag{26.84}$$

$$X_{zq}(p) = x_{zq} = x_L + x_q,$$
$$r_z = r + r_{\text{В}}, \tag{26.85}$$

$$G_{cd} = \frac{x_{af}}{r_f(1+pT_{d0})}. \tag{26.86}$$

The load on can be considered as the short circuit of the generator that possses the resistance r_z, $x_{zq,}$ and x_{zd}. Under the given conditions, the load on occurred at idle operatin, therefore

$$u_d(0) = 0, \quad u_q(0) = U_{m0};$$
$$\Delta i_d = i_d, \qquad \Delta i_q = i_q. \tag{26.87}$$

Before the generator short circuit equivalent to voltage application:

$$\Delta u_d + \Delta u_{zd} = 0,$$

$$\Delta u_q + \Delta u_{zq} = -U_{m0}.$$

At the same time (26.83) will be rewritten as:

$$r_z i_d(p) - \text{♟}_{ad} x_{zq} i_{zq}(p) = 0,$$
$$\text{♟}_{ad} X_{zd}(p) i_d(p) + r_z i_q(p) = -U_{m0} - \text{♟}_{ad} G_{cd} \Delta u_f(p). \tag{26.88}$$

From here

$$i_d(p) = -\frac{x_{zq}\text{♟}_z U_{m0} + x_{zq}\text{♟}^2{}_{ad} G_{cd}\Delta u_f(p)}{r_z^2 + x_{zq}\text{♟}^2{}_{ad} X_{zd}(p)},$$
$$i_q(p) = -\frac{r_z U_{m0} + r_z\text{♟}_{ad} G_{cd}\Delta U_f(p)}{r_z^2 + x_{zq}\text{♟}^2{}_{ad} X_{zd}(p)} \tag{26.89}$$

or

$$i_d(p) = i_{d1}(p) + i_{d2}(p),$$

$$i_q(p) = i_{q1}(p) + i_{q2}(p),$$

where

$$i_{d1}(p) = -\frac{x_{zq}\text{♟}_{ad}(1+pT_{d0})U_{m0}}{b_z(1+pT'_{zd})},$$
$$i_{q1}(p) = -\frac{r_z(1+pT_{d0})U_{m0}}{b_z(1+pT'_{zd})}, \tag{26.90}$$

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$$i_{d2}(p) = -\frac{x_{zq}\omega^2{}_{ad}G_{cd}(1+pT_{d0})\Delta u_f(p)}{b_z(1+pT'_{zd})},$$
$$i_{q2}(p) = -\frac{r_z\omega_{ad}G_{cd}(1+pT_{d0})\Delta u_f(p)}{b_z(1+pT'_{zd})}, \tag{26.91}$$

here:

$$T'_{zd} = \frac{r_z^2 + x_{zq}x'_{zd}\omega^2{}_{ad}}{r_z^2 + x_{zq}x_{zd}\omega^2{}_{ad}}T_{d0} = \frac{b_z'}{b_z}T_{d0},$$

$$b_z = (r+r_{\text{в}})^2 + (x_L + x_q)(x_L + x_d)\omega_r^2, \tag{26.92}$$

$$b_z' = (r+r_{\text{в}})^2 + (x_L + x_q)(x_L + x_d')\omega_r^2.$$

Excitement voltage is constant. If the excitment voltage under load on is constant, then

$$\Delta u_f(p) = 0 \text{ и } i_d(p) = i_{d1}(p), \quad i_q(p) = i_{q1}(p).$$

Equation (26.90) in its original form has the following form:

$$i_{d1} = -\omega_{ad}U_{m0}\frac{x_{rq}}{b_z}(1 + K_3 e^{-\frac{t}{T'_{zd}}}, \tag{26.93}$$

where

$$K_3 = \frac{b_z}{b_z'} - 1 = \frac{(x_L+x_q)(x_d-x_d')\omega^2{}_{ad}}{(r+r_{\text{в}})^2+(x_L+x_q)(x_L+x_d')\omega^2{}_{ad}}, \tag{26.94}$$

or

$$i_{d1} = -I_{d1} - (i'_{d1} - I_{d1})e^{-\frac{t}{T'_{zd}}},$$
$$i_{q1} = \frac{r_z}{x_{zq}\omega_{ad}}i'_{d1}, \tag{26.95}$$

where

$$I_{d1} = x_{zq}\omega_{ad}U_{m0}\frac{1}{b_z},$$
$$i'_{d1} = x_{zq}\omega_{ad}U_{m0}\frac{1}{b'_z}. \tag{26.96}$$

From (26.80) and (26.81) for the module i. в. of the load voltage vector we will find the expression [57-60]:

$$u_{m1} = \sqrt{(r_{\text{в}}i_{d1} - x_L\omega_{ad}i_{q1})^2 + (r_{\text{в}}i_{q1} + x_L\omega_{ad}i_{d1})^2} \tag{26.97}$$

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where $x_{ad} = 1$ and taking into account (26.95) we deduce:

$$u_{m1} = \frac{Z_{\text{в}} Z_{zq}}{x_{zq}} i_{d1}, \tag{26.98}$$

where

$$Z_{\text{в}} = \sqrt{r_{\text{в}}^2 + x^2{}_L}$$
$$Z_{zq} = \sqrt{(r + r_{\text{в}})^2 + \left(x_L + x_q\right)^2} \tag{26.99}$$

as $t = 0$ according to (26.95) and (26.98)

$$I_{dt0} = i'_{d1}, \quad U_{t0} = K_4' U_{m0} \,,$$

where

$$K_4' = \frac{Z_{\text{в}} Z_{zq}}{r_Z^2 + x_{zq} x'_{zd}} \tag{26.100}$$

If neglecting the active stator resistance and the load, then at *t = 0* according to (26.95), (26.96), (26.85) and (26.98) we deduce:

$$I_{dt0} = i'_{d1} = \frac{U_{m0}}{x_L + x_d'}$$

$$U_{t0} = \frac{x_L}{x_L + x_d'} U_{m0} = \frac{x_d' + x_L - x_d'}{x_L + x_d'} U_{m0} \,,$$

or

$$U_{t0} = U_{m0} - x_d' i'_{d1}. \tag{26.101}$$

Instantaneous voltage failure at load on

$$\Delta U_{t0} = \frac{x_d'}{x_L + x_d'} U_{m0} = x_d' i'_{d1} \,. \tag{26.102}$$

These formulas are identical as the previously deduced (26.23) and (26.24).

If a purely active load is enabled, that is, if $x_L = 0$, then putting *r = 0* is deduced

$$U_{t0} = \frac{r_{\text{в}} \sqrt{r_{\text{в}}^2 + x_q^2}}{r_{\text{в}}^2 + x_q x_d'} U_{m0} \,. \tag{26.103}$$

If in в.о. $r_{*\text{в}} = 1$ and $x_{*q} = 0{,}7$, then

$$U_{t0} = \frac{1{,}22}{1 + 0{,}7 x_d'} U_{m0} \,. \tag{26.104}$$

Thus, in some cases, in our example, at transient inductive resistances in o. e. less than 0.31, the the active load on may be accompanied by an instantaneous increase in the voltage. Further, the voltage will decrease to the set value

$$U_1 = K_4 U_{m0},$$

where

$$K_4 = \frac{Z_{\text{в}} Z_{zq}}{r_z^2 + x_{zq} x_{zd}}.$$

If $= r_{\text{в}} = 0$, we will deduce for U_1 expression (26. 23).

If $x_L \approx 0$ and $r \approx 0$, we will deduce

$$U_1 \approx \frac{r_{\text{в}}\sqrt{r_{\text{в}}^2 + x_q^2}}{r_{\text{в}}^2 + x_q x_d} U_{t0}. \qquad (26.105)$$

The excitation voltage increases according to the rectilinear law. In the presence of a voltage controller, as it follows from (26.89), in stator currents, in addition to the components determined by (26.90) and (26.93), there will further be components determined by (26.91).

If the excitation voltage changes by the law as follws

$$u_f = U_{f0} + kU_{f0} t,$$

then

$$\Delta u_f = kU_{f0} t$$

and

$$\Delta u_f(p) = \frac{kU_{f0}}{p}. \qquad (26.106)$$

Then taking into account (26. 86) and (26. 2):

$$i_{d2}(p) = -\frac{k x_{za} \mu^2{}_{ad} U_{m0}}{(1 + pT'_{zd}) p\, b_z},$$

$$i_{q2}(p) = -\frac{k r_z \mu_{ad} U_{m0}}{(1 + pT'_{zd}) p\, b_z} \qquad (26.107)$$

From here

$$i_{zd} = -k x_{zq} \mu^2{}_{ad} U_{m0} \left[t - T'_{zd} (1 - e^{-\frac{t'}{T'_{zd}}} \right];$$

$$i_{zq} = \frac{r_z}{x_{zq} \mu_{ad}} i_{d2}, \qquad (26.108)$$

where $t' = t - t_1$, and t_1 is the voltage controller action time [60-62].

Module i. в. of the machine voltage due to the controller action will be determined by the formula similar to (26.98)

$$u_{m2} = \frac{z_{\text{в}} z_{zq}}{x_{zq}} i_{d2}. \qquad (26.109)$$

The module of entire i. в. of the machine voltage equals

$$u_m = u_{m1} + u_{m2} = u_m = \frac{z_{\text{в}} z_{zq}}{x_{zq}} (i_{d1} + i_{d2}) \qquad (26.110)$$

or

$$u_m = K_4 \varkappa_{ad} U_{m0} \left\{ \left(1 + K_3 e^{-\frac{t'}{T'_{zd}}}\right) + k \varkappa_{ad} \left[t' - -T'_{zd} \left(1 - e^{-\frac{t'}{T'_{zd}}}\right)\right]\right\}. \qquad (26.111)$$

Equating the derivative of the right side of the last expression to zero and solving the resulting equation, we find the time of the minimum voltage onset

$$t_m = T'_{zd} \ln \left(\frac{K_3}{kT'_{zd}} + e^{-\frac{t'}{T'_{zd}}} \right). \qquad (26.112)$$

Substituting this expression into (26.111) for the minimum of machine voltage at $\varkappa_{ad} = 1,$ we deduce

$$u_{min} = \left\{ 1 + \left[k \left(T'_{zd} \ln \frac{K_3}{kT'_{zd}} + e^{-\frac{t'}{T'_{zd}}} \right) - t_1 \right] \right\} \frac{z_{\text{в}} z_{zq}}{b_z} U_{m0}. \qquad (26.113)$$

Excitation voltage increases by exponential law. Let the increase in the excitation voltage be expressed by the equation

$$\Delta u_f = \Delta U_f (1 - e^{-\frac{t}{T_{\text{в}}}}) \qquad (26.114)$$

where ΔU_f defines the voltage limits equal to

$$U_f = U_{f0} + \Delta U_f, \qquad (26.115)$$

$T_{\text{в}}$ is the time constant of the curve of exciter voltage build-up.

In the operator form equation (26.114) has the following form

$$\Delta u_f = \Delta U_f \frac{1}{1 + pT_{\text{в}}}. \qquad (26.116)$$

Substitution of this value Δu_f into (26. 91) gives

$$i_{d2}(p) = - \frac{x_{zq} \varkappa^2{}_{ad} \Delta E_m}{(1 + pT'_{zd})(1 + pT_{\text{в}}) b_z}, \qquad (26.117)$$

where

$$\Delta E_m = \frac{x_{af} \Delta U_f}{r_f};$$

From here

$$i_{d2} = -x_{zq}\mu^2{}_{ad}\Delta E_m \frac{1}{b_z}\left(1 - \frac{T'_{zd}e^{-\frac{t}{T'zd}} - T_{\text{в}}e^{-\frac{t}{T_{\text{в}}}}}{T'_{zd} - T_{\text{в}}}\right). \qquad (26.118)$$

Entire longitudinal current at $\mu_{ad} = 1$

$$i_d = i_{d1} + i_{d2}\ .$$

Formula (26.98) is also valid for entire voltages and the current, i.e.

$$u_m = \frac{Z_{\text{в}}Z_{zq}}{x_{zq}} i_d\ , \qquad (26.119)$$

or

$$u_m = K_4\mu_{ad}\left[\left(1 + K_3 e^{-\frac{t}{T'zd}}\right)U_{m0} + + \left(1 - \frac{T'_{zd}e^{-\frac{t}{T'zd}} - T_{\text{в}}e^{-\frac{t}{T_{\text{в}}}}}{T'_{zd} - T_{\text{в}}}\right)\mu_{ad}\Delta E_m\right]. \qquad (26.120)$$

Time to achieve the minimum voltage

$$t_m = \frac{T_{\text{в}}T'_{zd}}{T'_{zd} - T_{\text{в}}}\left\{\frac{t_1}{T_{\text{в}}} - \ln\left[e^{\frac{t_1}{T'zd}} - K_3\frac{U_{m0}}{\Delta E_\infty}\left(\frac{T'_{zd} - T_{\text{в}}}{T'_{zd}}\right)\right]\right\}. \qquad (26.121)$$

The generator is self-excited. By the proper choice of the circuit and elements of self-excitation system of the synchronous generator, it is possible to achieve with greater or lesser accuracy the direct proportionality between the excitation voltage U_f and E. M. F. E_m, induced in the stator by the excitation flow, that is, we can deduce

$$U_f \approx k_u E_m. \qquad (26.122)$$

This provides the best way to maintain the voltage constant or the specified staticity of the external characteristics of synchronous generator.

From the vector diagram of a nonsalient pole synchronous machine and formula (4.1) it should be

$$E = U_q - x_d I_d - rI_q. \qquad (26.123)$$

Taking one law of excitation votage regulation of the excitation voltage in steady and transitional modes, neglecting the value of rI_q in the last formula and substituting it into (26.122), we deduce:

$$u_f = k_u u_q - k_i i_d, \qquad (26.124)$$

where

$$k_i = k_u x_d. \qquad (26.125)$$

At idle operation, the excitation voltage

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$$U_{f0} = k_u U_{m0}\,, \tag{26.126}$$

where $U_{m0}$is the stator voltage amplitude at idle operation.

Increasing the excitation voltage at load on

$$\Delta u_f = u_f - U_{f0} = k_u u_q - k_i i_d - k_u U_{m0}. \tag{26.127}$$

According to (89. 4) I.B. of the machine voltage $\underset{u}{\rightarrow}$ equals and is exactly opposite to i. b. of the load voltage $\underset{u_z}{\rightarrow}$, therefore

$$u_q = -u_{zq}$$

and

$$\Delta u_f = -\left(U_{f0} + k_u u_{zq} + k_i i_d\right). \tag{26.128}$$

Substitutition of the value u_{zq} from (26.80) into (26.128), taking into account (26.81), gives

$$\Delta u_f(p) = -\left[U_{f0} + k_{ui} i_d(p) + k_u r_{\text{в}} i_q(p)\right], \tag{26.129}$$

where

$$k_{ui} = k_i + k_u x_L \lambda_{ad}\,. \tag{26.130}$$

Substituting the values $\Delta u_f(p)$, $x_{zd}(p)$ and G_{cd} from (26.129), (26.84) and (62.35) into (26.88) and solving the deduced system with respect to currents, we have:

$$\begin{aligned} i_d(p) &= i_{d1}(p) + i_{d2}(p), \\ i_q(p) &= i_{q1}(p) + i_{q2}(p) \end{aligned} \tag{26.131}$$

where

$$\begin{aligned} i_{d1}(p) &= -\frac{x_{zq} U_{m0} \lambda_{ad}(1 + pT_{d0})}{(\alpha_{\text{св}} + pT'_{zd}) b_z}, \\ i_{d2}(p) &= \frac{x_{zq} \lambda^2{}_{ad} U_{m0}}{(\alpha_{\text{св}} + pT'_{zd}) b_z}, \end{aligned} \tag{26.132}$$

$$\begin{aligned} i_{q1}(p) &= -\frac{(1 + pT_{d0}) r_z U_{m0}}{(\alpha_{\text{св}} + pT'_{zd}) b_z} \\ i_{q2}(p) &= \frac{r_z \lambda_{ad} U_{m0}}{(\alpha_{\text{св}} + pT'_{zd}) b_z} \end{aligned} \tag{26.133}$$

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Inputs into equations for currents of magnitude b_z, T'_{zd} and $\alpha_{с\text{в}}$ are determined from the following expressions:

$$b_z = r_z^2 + x_{zq} x_{zd} \omega^2{}_{ad},$$
$$b_z' = r_z^2 + x_{zq} x'_{zd} \omega^2{}_{ad} \tag{26.134}$$

$$T'_{zd} = \frac{b_z'}{b_z} T_{d0}\ , \tag{26.135}$$

$$\alpha_{с\text{в}} = 1 - \frac{x_{af}}{b_z r_f}\left(k_u r_{\text{в}} r_z \omega_{ad} + k_{ui} x_{zq} \omega^2{}_{ad}\right). \tag{26.136}$$

Let's determine $\alpha_{с\text{в}}$ as the self-excitation coefficient; at $k_u = k_i = 0$ we obtain $\alpha_{с\text{в}} = 1$.

Expanding the operator expressions (26.132) and (26.133), we deduce the following equations for currents:

$$i_{d1} = -\frac{x_{zq}\omega_{ad} U_{m0}}{b_z}\left(\frac{1}{\alpha_{с\text{в}}} + K_{с\text{в}} e^{-\frac{\alpha_{с\text{в}}}{T'_{zd}}t}\right),$$
$$i_{d2} = \frac{x_{zq}\omega_{ad} U_{m0}}{b_z \alpha_{с\text{в}}}\left(1 - e^{-\frac{\alpha_{с\text{в}}}{T'_{zd}}t}\right). \tag{26.137}$$

$$i_{q1} = -\frac{r_z U_{m0}}{b_z}\left(1 + K_{с\text{в}} e^{-\frac{\alpha_{с\text{в}}}{T'_{zd}}t}\right),$$
$$i_{q2} = \frac{r_z \omega_{ad} U_{m0}}{b_z \alpha_{с\text{в}}}\left(1 - e^{-\frac{\alpha_{с\text{в}}}{T'_{zd}}t}\right), \tag{26.138}$$

where

$$K_{с\text{в}} = \frac{T_{d0}}{T'_{zd}} - \frac{1}{\alpha_{с\text{в}}} = \frac{b_z}{b_z'} - \frac{1}{\alpha_{с\text{в}}}\ . \tag{26.139}$$

From the comparison (26.137) and (26.138) should be:

$$i_{q1} = \frac{r_z}{x_{zq}\omega_{ad}} i_{d1}\ , \tag{26.140}$$

$$i_{q2} = \frac{r_z}{x_{zq}\omega_{ad}} i_{d2}. \tag{26.141}$$

Module i, b. of the load voltage in accordance with (26.80)

$$u_m = \sqrt{u^2{}_{zq} + u^2{}_{zd}} = \sqrt{\left(r_{\text{в}} i_d - x_L \omega_{ad} i_q\right)^2 + \left(r_{\text{в}} i_q + x_L \omega_{ad} i_d\right)^2}. \tag{26.142}$$

Substituting here the values of currents from (26.131) (26.140) and (26.141), after a series of transformations we deduce

$$u_m = \frac{Z_{\text{в}} Z_{zq}}{x_{zq}\nu_{ap}}(i_{d1} + i_{d2}), \tag{26.143}$$

where $Z_{\text{в}} Z_{zq}$ are determined (26.99)

From (26.131) and (26.137) at synchronous rotor spinning velocity, that is at $\nu_{ad} = 1$ for the entire longitudinal current we have

$$i_d = -\frac{x_{zq} U_{m0}}{b_z{}'} e^{-\frac{\alpha_{\text{св}}}{T'_{zd}} t}. \tag{26.144}$$

Substituting this value of current into (26.143), we obtain at $\nu_{ad} = 1$, the following equations for the amplitude of the machine voltage:

$$u_m = \frac{Z_{\text{в}} Z_{zq}}{b_z{}'} U_{m0} e^{-\frac{\alpha_{\text{св}}}{T'_{zd}} t}, \tag{26.145}$$

or

$$u_m = \frac{\sqrt{r_{\text{в}}^2 + x_L^2}\sqrt{(r_{\text{в}} + r)^2 + \left(x_q + x_L\right)^2}}{(r_{\text{в}} + r)^2 + (x_q + x_L)(x'_d + x_L)} U_{m0} e^{-\frac{\alpha_{\text{св}}}{T'_{zd}} t}. \tag{26.146}$$

At the initial moment, the voltage decreases to the magnitude

$$U_{t0} = \frac{\sqrt{r_{\text{в}}^2 + x_L^2}\sqrt{(r_{\text{в}} + r)^2 + \left(x_q + x_L\right)^2}}{(r_{\text{в}} + r)^2 + (x_q + x_L)(x'_d + x_L)} U_{m0}. \tag{26.147}$$

Initial voltage failure

$$\Delta U_{t0} = U_{m0} - U_{t0} = \left(1 - \frac{Z_{\text{в}} Z_{zq}}{b_z{}'}\right) U_{m0}. \tag{26.148}$$

If $\frac{\alpha_{\text{св}}}{T'_{zd}} > 0$ and, thus $\alpha_{\text{св}} > 0$, that is, if the coefficients k_u and k_i are ultimately small, and compounding is weak, then u_m will change according to curve 1. Fig. 26.3. Changing at the initial instant of switching on almost instantaneously by the value ΔU_{t0}, then the voltage drops to zero with the time constant $\frac{T'_{zd}}{\alpha_{\text{св}}}$.

If $\alpha_{\text{св}} < 0$, that is, if k_u and $k_i$are sufficiently high and the compounding is powerful enough, then the network voltage in the absence of saturation will change in curve 2, increasing infinitely. In the presence of saturation, the voltage in this case will increase in curve 3, seeking a stable final value [63].

If $\alpha_{\text{св}} = 0$ and, consequently,

$$\frac{b_z r_f}{x_{af}} = k_u r_{\text{в}} r_z + (k_i + k_u x_L) x_q, \tag{26.149}$$

That voltage will be constant and will be equal to U_{t0} (.26. 3, line 4).

Let's consider two separate cases.

1. If there is a purely inductive load on, then, substituting $r_z = r_{\text{в}} + r = 0$ into the previous equation, we deduce

$$i_m = -i_d = \frac{U_{m0}}{x_d' + x_L} e^{-\frac{t}{T'_{ui}}}, \tag{26.150}$$

$$u_m = \frac{x_L}{x_d' + x_L} e^{-\frac{t}{T'_{ui}}} \tag{26.151}$$

where

$$T_{ui} = \frac{x_L + x_d'}{x_L + x_d - \frac{k_{ui} x_{af}}{r_f}} T_{d0}\ . \tag{26.152}$$

The value of current and voltage at the initial moment:

$$I_{t0} = \frac{U_{m0}}{x_L + x_d'}, \tag{26.153}$$

$$U_{t0} = \frac{x_L}{x_L + x_d'} U_{m0} = U_{m0} - x_d' I_{t0}. \tag{26.154}$$

Initial voltage failure at load on

$$\Delta U_{t0} = \frac{x_d'}{x_L + x_d'} U_{m0} = x_d' I_{t0}. \tag{26.155}$$

2. If there is a purely active load on, that is if $x_L = 0$, then, putting into (26.147) $r = 0$, we obtain for the voltage at the initial instant the expression similar to the one previously deduced (26.103).

If in relative units $r_{*\text{в}} = 1$ and $x_{*q} = 0{,}7$, so for U_{t0} we deduce the expression (26.104).

Thus, as with a constant voltage of excitation in self-excited generators, in some cases the active load on may be accompanied by an instantaneous increase in voltage [64]. In the future, the voltage will decrease or increase, depending on the degree of compensation according to the sign and magnitude $\alpha_{\text{св}}$.

2.26. Algorithm for designing synchronous machines with permanent magnets

Let's consider some variants of synchronous generators design excitable by permanent magnets for autonomous electric power plants (AEPP). Calculation algorithms of respective generators possess a number of features in comparison with the known methods. The main of them is related to the search for the best options according to the technical requirements for designing AEPP broadly as a single system. In this case, as a rule, a considerable number of estimated variants of the machine with different constructive performances is considered and the most rational ones are selected from them for further in-depth processing (for example, with the use of the corresponding CAD).

It is, therefore, an accelerated calculation of the main indicators of machines, taking into account both traditional and specific constraints dictated by the harmonization of machne characteristics and AEPP in general. Of course, preliminary estimates of the main machine indicators are to be supplemented and refined on the basis of additional calculations of the electromagnetic, strength and thermal characteristics of the macine on the basis of traditional methods.

Since the calculation algorithms should be relatively universal, a large role in them is given to dimensionless parameters and indicators that have a visual physical content and can be selected based on existing recommendations (far from complete for AEPP electric machines) and on the basis of rather obvious intuitive engineering representations .

Let's consider the features of the calculated algorithms of machines with PM in relation to synchronous generators. The calculation of synchronous motors can be performed on a similar basis. Let it be necessary to carry out an approximate calculation of the synchronous generator with PM on the given nominal values of power $S_{\text{ном}}$ of the number of phases *m,* voltage $U_{\text{ ном}}$ of the power factor $cos\ \varphi$ PM parameters are supposed to be known *(*$\text{B}_r, \text{H}_c, \alpha, \mu_{\text{в}}$*)*.

It is possible to allocate four schemes of machine design for the imposed capacity depending on the specification requirements.

In the first calculation scheme, the generator is designed under a known primary drive with a fixed rotational speed *n* and is expected to produce the current of given frequency *f*.

The second design scheme involves the operation of a generator with a maximum permissible speed of the rotor u_{max} and in accordance with the limiting mechanical loads. The task u_{max} is related to minimizing the size and mass of the generator.

The third calculation scheme, besides the task V_{max}, involves fixing the electromechanical constant T_j, which characterizes the launch speed of the installation.

The current frequency *f* in the second and third circuits is not strictly limited, since in some cases AEPPs can operate with variables *f* (for example, in heating, lighting and other loads) or contain consumers calculated for the frequency that is determined during the design of the generator.

In the fourth design scheme of the machine the values *n* and T_j are given. In order to find the main dimensions of the generator in all cases, first they determine the value E_0^*. In general, they can be calculated according to (2.23), (2.24) on the basis of the relative magnetic conductors ЛЛΛ_δ^*, and the parameters a and $\mu_{\text{в}}^*$^. At the same time, the number of pole pairs is calculated as well-known. If the calculation is based on the first scheme, then $\text{p} = 60f/n$. When using the second and third calculation schemes, p is first given for general considerations, and then, if necessary, p_{opt} is found by additional calculations (see §2.7).

Let us first consider the generator with tangentially magnetized PM on the basis of REM (Fig. 2.33, a), which is used in AEPPs from tens to hundreds of kilowatts. In this case $\mu_{\text{в}}^* =$ 1= 1, $a = 0$ and the calculation of E_0^* and $1_{\text{К}}{}^*$ is carried out according to (2.30) and (2.31). We first determine the relative magnitudes of the magnets for a fixed value of p, assuming that the diameter d of the inner sleeve, to which the magnets adjoin, exceeds by Δ the diameter d, on which the extension of the sides of neighboring magnets intersects. The relative index $\Delta^*= \Delta/D$ takes into account the technological factors (usually $\Delta^*\approx 0{,}1 \dots 0{,}2$). Since the nonmagnetic insert of the outer cylinder adjacent to the magnet has approximately the same tangential dimension as the magnet, then $\pi D - \pi d' \approx 2p\alpha_p\tau$, from where $d' \approx (1 - \alpha_p)D$ is the constructive coefficient of pole overlay (usually $\alpha_p \approx 0{,}6 \dots 0{,}7$ for magnets based on REM).

Taking into account the ratio $d = d' + \Delta = D\left(1 - \alpha_p + \Delta^*\right)$, the radial height of the magnet is $b_M = 0{,}5(D - d) = 0{,}5\left(\alpha_p - \Delta^*\right)D$.

Relative dimensions of the magnet are the height and tangential length, respectively

$$b_M^* = b_M/(0{,}5D) = \alpha_p - \Delta^* \tag{27.1}$$

$$L^* = 21_M/b_M = \left(1 - \alpha_p\right)\tau/b_M = \frac{\pi\left(1 - \alpha_p\right)}{p\left(\alpha_p - \Delta^*\right)}$$

The basic values for b_M and L^* are selected n such a way to give a clear idea of the form and location of PM. When determining b_M^*, the value Δ^* may indirectly take into account the thickness of the outer cylinder or bandage.

Th coefficient of rotor filling by magnets

$$r_{эм} = 2p(2l_M)b_M/(2\pi D^2/4) = 2\left(1 - \alpha_p\right)\left(\alpha_p - \Delta^*\right) \tag{27.2}$$

Now we can proceed to finding Λ_δ^*, Λ_a^*, Λ_{oa}^*. Relative conductivity of the gap (per one pole of the magnet), taking into account the scales

$\Delta_{в}^*= (0{,}5\mu_0\alpha_\delta\tau l/\delta'')H_c l_M/(B_r b_M l)$, where the factor 0.5 takes into account that one pole of the magnet accounts for half of the above mentioned.

After performing the necessary substitutions, we deduce

$$\Lambda_\delta^* = \alpha_\delta\pi L^*/\left(8p\mu_r^*\delta^{*''}\right) = \frac{\alpha_\delta\pi^2\left(1 - \alpha_p\right)}{8\left(\alpha_p - \Delta^*\right)\mu_r^*\delta^{*''}p^2}$$

where α_δ is the calculated coefficient of pole overlap;

$\mu_r^* = \frac{B_r}{(\mu_0 H_c)}$ is the relative permeability; $\delta^{*''} = \frac{\delta''}{D}$ is the relative working gap; $\delta'' = r_\delta r_\mu\delta$ is the constructive clearance.

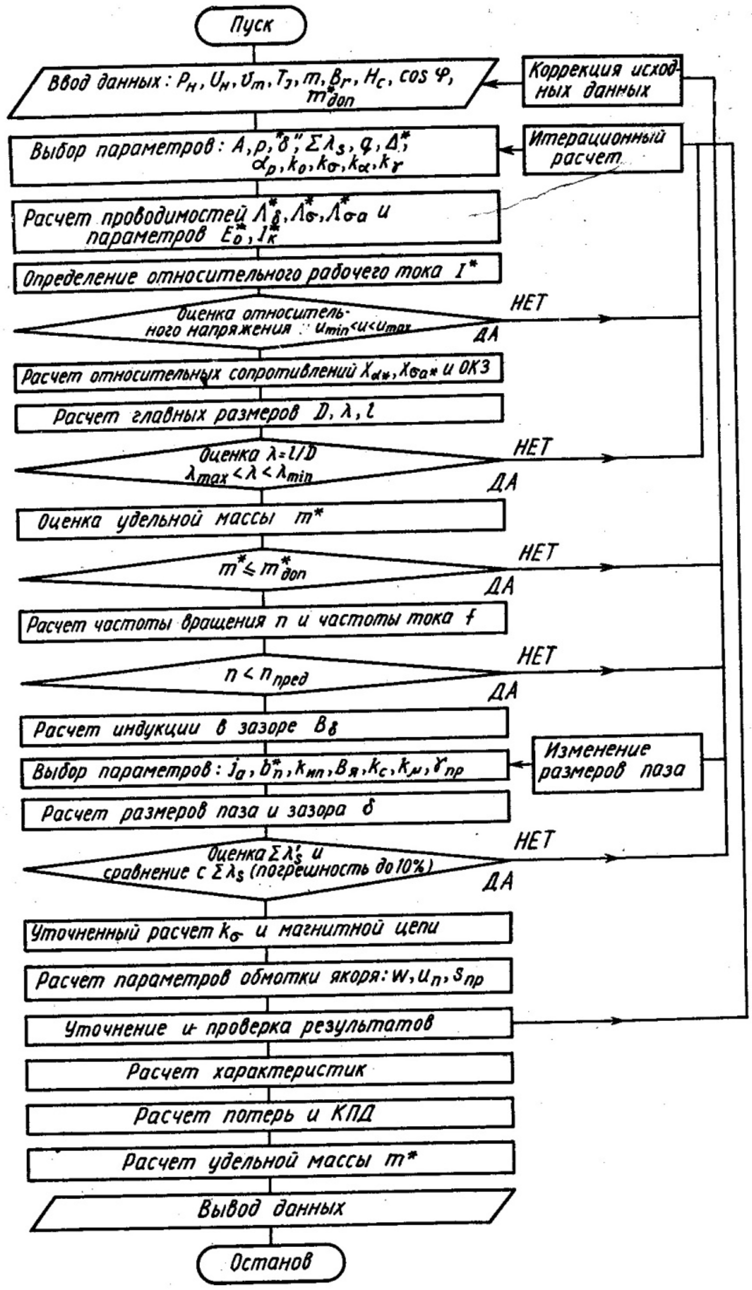

Figure 27.1 - Scheme of the algorithm for designing a synchronous motor with permanent magnets.

Conclusions

The monograph considered and presented various versions of electric motors with permanent magnets. A comparative characteristic of electric motors with permanent magnets and already studied synchronous machines has been carried out. The mathematical apparatus was involved in the work, which reflects the operation of engines with different parameters and in different modes. The calculations performed using the mathematical apparatus showed that these types of electric motors deserve increased attention and their further application in electric transport, as traction motors. The obvious conclusion from the mathematical calculations made is that this type of engine is more economical, with high efficiency, requires less time and maintenance costs, and also has a longer service life.

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MONOGRAPH

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